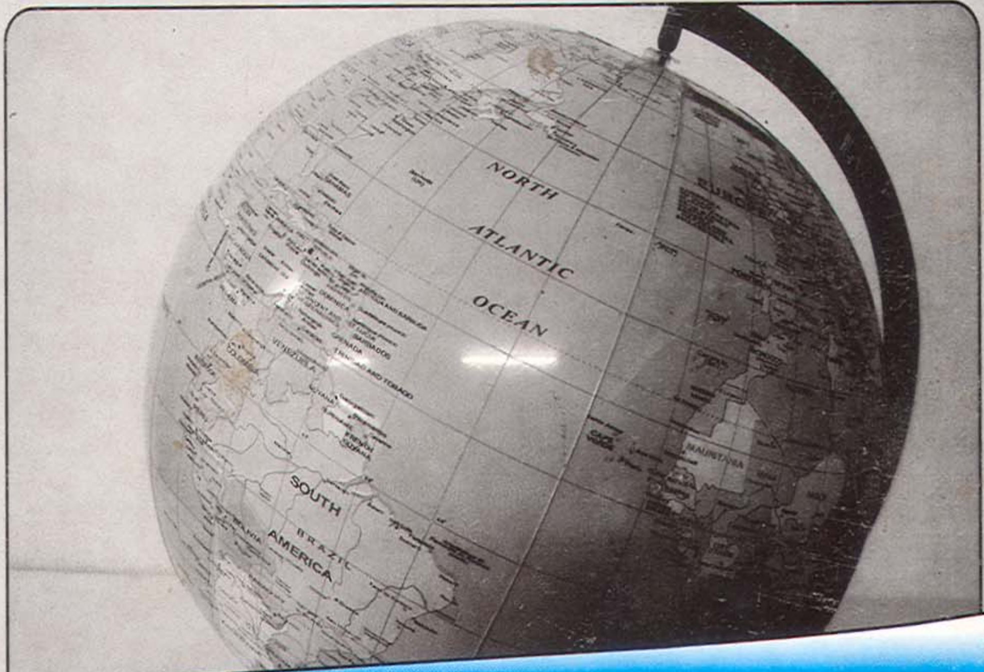
Secondary

MATHEMATICS

Form Four Teachers' Guide

Third Edition



The two towns, Macapa, Brazil (0°, 51°W) and Nanyuki, Kenya (0°, 37°E) are on the same latitude, i.e., Equator. Since they lie on a great circle, the distance between them is...

New Syllabus

KENYA LITERATURE BUREAU

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Chapter One

MATRICES AND TRANSFORMATIONS

The learner met matrices in Book Three. Some transformations were also dealt with in Book Two. A review of the transformations and operations involving matrices is therefore necessary. In this topic, matrices will be used to carry out transformations.

Objectives

By the end of the topic, the learner should be able to:

(i) relate image and object under a given transformation on the cartesian plane.
(ii) determine the matrix of a transformation.
(iii) perform successive transformations.
(iv) determine and identify a single matrix for successive transformations.
(v) relate identity matrix and transformation.
(vi) determine the inverse of a transformation.
(vii) establish and use the relationship between area scale factor and determinant of a matrix.
(viii) determine shear and stretch transformation.
(ix) define and distinguish isometric and non-isometric transformations.
(x) apply transformation to real life situations.

Time: Twenty one lessons.

Teaching/Learning Activities

Matrices of Transformation

- The teacher should lead the learner to revise the position vectors of given points in the cartesian plane, as in the students' book.
- The learner should be led to find the position vectors of an image point using a transformation matrix, as in the students' book.
- The learner should be involved in relating the object and the image under a transformation, as in figure 1.1.
- The learner to do exercise 1.1.

Finding the Matrix of a Transformation

- The learner should be guided in finding the matrix of a transformation, as illustrated in the students' book.
- The teacher should guide the learner through example 3.
- The learner to do exercise 1.2.
- The teacher should discuss the use of a unit square in finding the matrix of transformation, as in the students' book.
- The learner should be led through examples 4, 5, 6, 7 and 8.
- The learner to do exercise 1.3.

Successive Transformations

- The teacher should introduce successive transformations as in the students' book.
- The learner should be led to perform successive transformations,as in examples 9 and 10.
- The learner should be guided to obtain a single matrix for successive transformations, as in examples 11 and 12.
- The learner to do exercise 1.4.

Inverse of a Transformation

- The teacher should introduce the inverse of a transformation, as in the students' book.
- The learner should be led to find the inverse of a transformation, as in example 13.

Area Scale Factor and Determinant of a Matrix

- The teacher should lead the learner to establish the relationship between area scale factor and the determinant of a transformation matrix, as in the students' book.
- The learner to do exercise1.5.

Shear and Stretch

- The teacher should discuss shear as in the students' book.
- The teacher should guide the learner through examples 14, 15 and 16.
- The teacher should discuss stretch, as in the students' book.
- The learner should be taken through example 17.

Isometric and Non-isometric Transformations

- The teacher should lead the learner through the definition of isometric and non-isometric trasformations, as in the students' book.

- The learner should b[illegible]ed to identify isometric and non-isometric transformations.
- The learner to do exercise 1.6.

Additional Hints

The teacher should make use of peg-boards and rubber bands to demonstrate shear, stretch and enlargement.

Evaluation

The learner should be given a written test on matrices and transformations.

Answers

Exercise 1.1

1. (a) A′ (2, -3) B′ (2, -5) C′ (6, -5) D′ (6, -3)
   Reflection in the line y = 0 or x-axis.

   (b) A′ (-2, -3) B′ (-2, -5) C′ (-6, -5) D′ (-6, -3)
   Half-turn about (0, 0).

   (c) A′ (3, 2) B′ (5, 2) C′ (5, 6) D′ (3, 6)
   Reflection in the line y = x.

   (d) A′ (3, -2) B′ (5, -2) C′ (5, -6) D′ (3, -6)
   Negative quarter turn about (0, 0) or positive three quarter turn about (0, 0).

   (e) A′ (4, 6) B′ (4, 10) C′ (12, 10) D′ (12, 6)
   Enlargement, scale factor 2, centre (0, 0).

   (f) A′ (-6, -9) B′ (-6, -15) C′ (-18, -15) D′(-18, -9)
   Enlargement, scale factor -3 centre (0, 0).

2. (a) Reflection in the y-axis or x = 0
   (b) Enlargement, centre (0, 0), scale factor 2.5
   (c) Positive quarter turn about (0, 0).
   (d) Enlargement, centre (0, 0), scale factor -0.5.
   (e) Reflection in the line y + x = 0.
   (f) Identity transformation.

3. Half-turn about the origin.

4. (0, 0); No.

5. Reflection in the line $y = \frac{1}{2}x$.

6. Reflection in the line $y = \frac{1}{3}x$.

7. Reflection in the line $y = -\frac{1}{4}x$.

8. (a) Positive 30° turn about (0, 0)
   (b) Positive quarter turn about (0, 0)
   (c) Negative 210° turn or positive 150° turn about (0, 0)
   (d) Negative quarter turn about (0, 0).

9. (a) Reflection in the line $y = \frac{-2}{3}x$
   (b) Reflection in the line $y = \frac{-5}{6}x$.

10. (a) Rotation through +30° about (0, 0).
    (b) Rotation through Θ° about (0, 0).

Exercise 1.2

1. (a) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ Reflection in the line $y + x = 0$.

   (b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Positive quarter turn about (0, 0).

   (c) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ Half turn about (0, 0).

   (d) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ Enlargement, scale factor $\frac{1}{2}$, centre (0, 0).

(e) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity transformation.

2. (a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (b) Translation $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d) Translation $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

(e) $\begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$

3. (a) $\begin{pmatrix} \frac{7}{25} & \frac{24}{25} \\ \frac{24}{25} & \frac{-7}{25} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{9}{41} & \frac{-40}{41} \\ \frac{-40}{41} & \frac{-9}{41} \end{pmatrix}$

4. (a) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$

(c) $\begin{pmatrix} 0.7 & -0.7 \\ 0.7 & 0.7 \end{pmatrix}$ or $\begin{pmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{pmatrix}$

(d) $\begin{pmatrix} 0.7 & 0.7 \\ -0.7 & 0.7 \end{pmatrix}$ or $\begin{pmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{pmatrix}$

5. $\begin{pmatrix} \frac{-3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$; Reflection in the line y = 2x.

6. $\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$

7. Enlargement, centre (0, 0), scale factor -2. $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

8. Reflection in the line x = 0 or y-axis $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

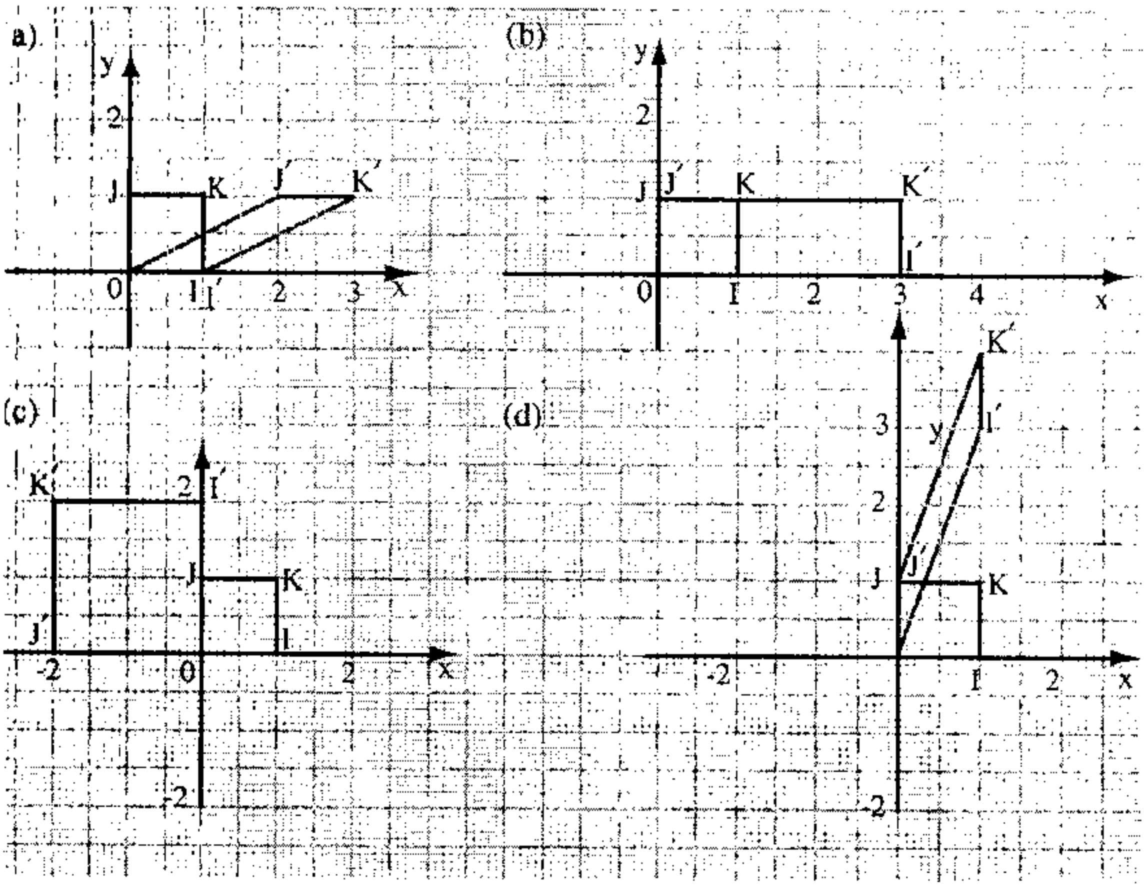
Exercise 1.3

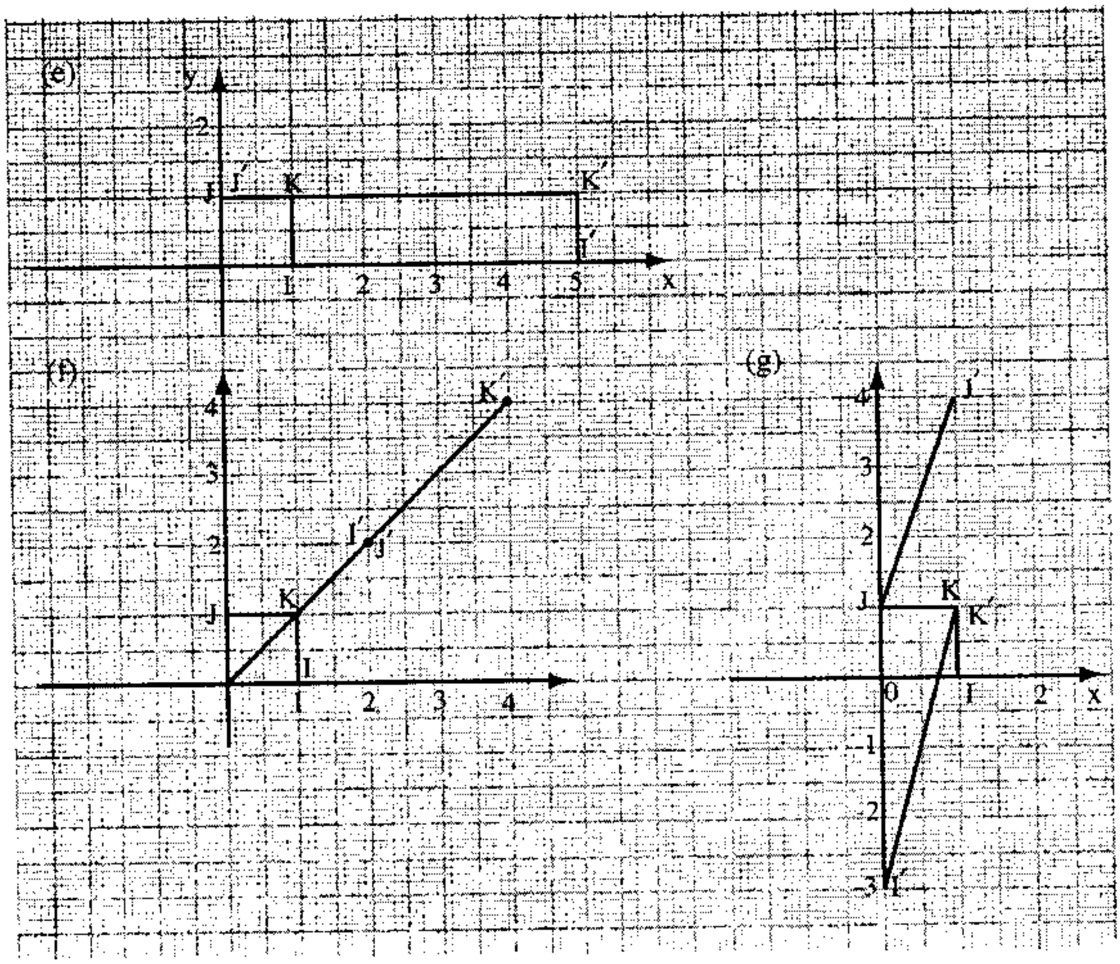
1. (a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(g) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (h) $\begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix}$ (i) $\begin{pmatrix} -2.5 & 0 \\ 0 & -2.5 \end{pmatrix}$

2.




(e)
y
2
J
J′
K
K′
I′
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J′
J
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2
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x
-1
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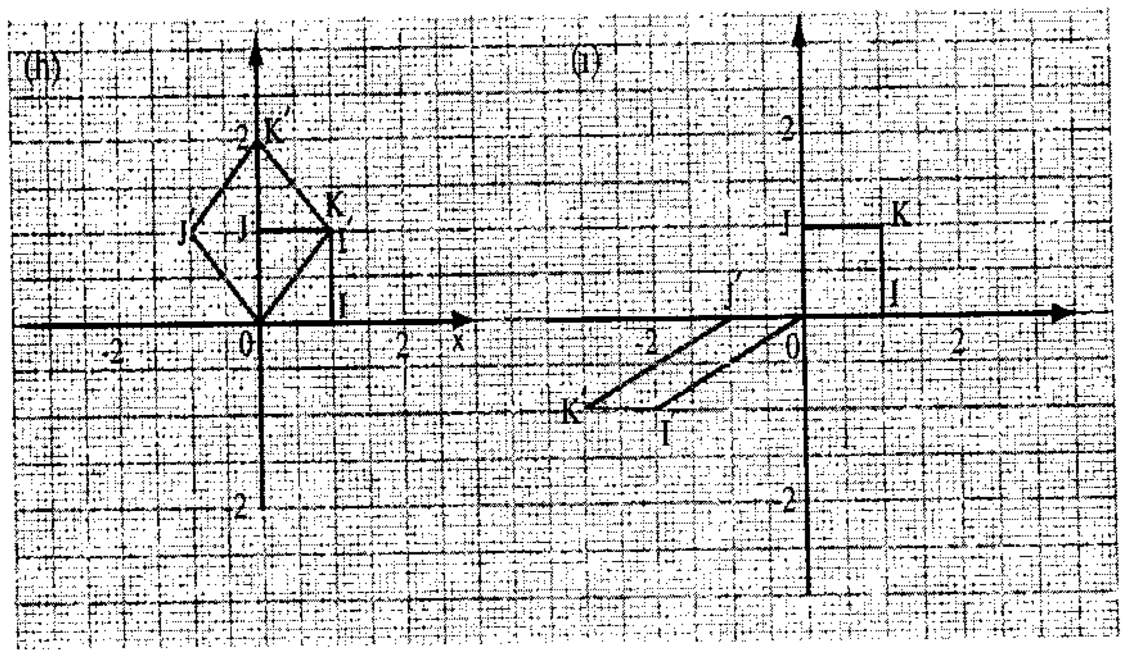
I′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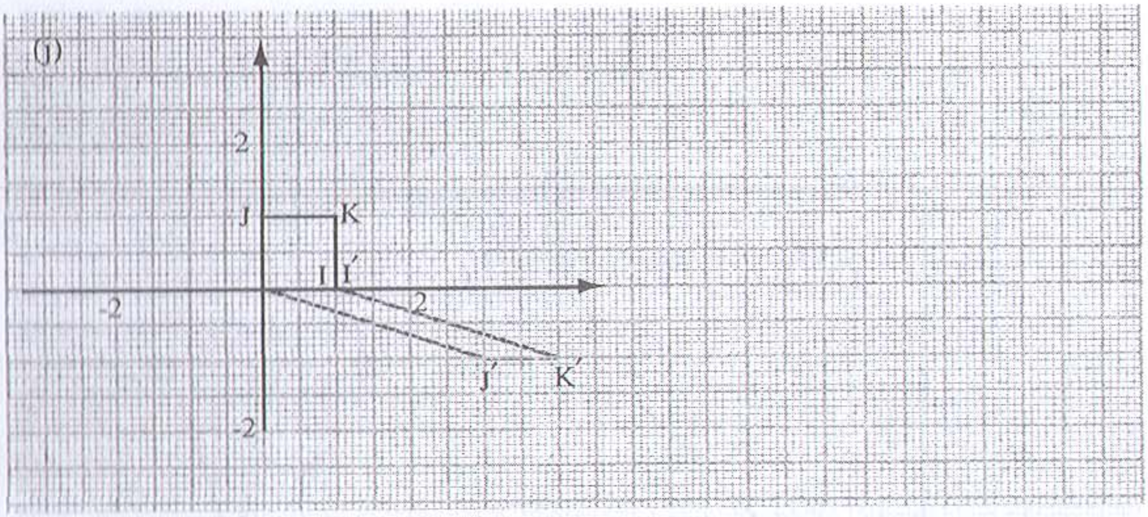
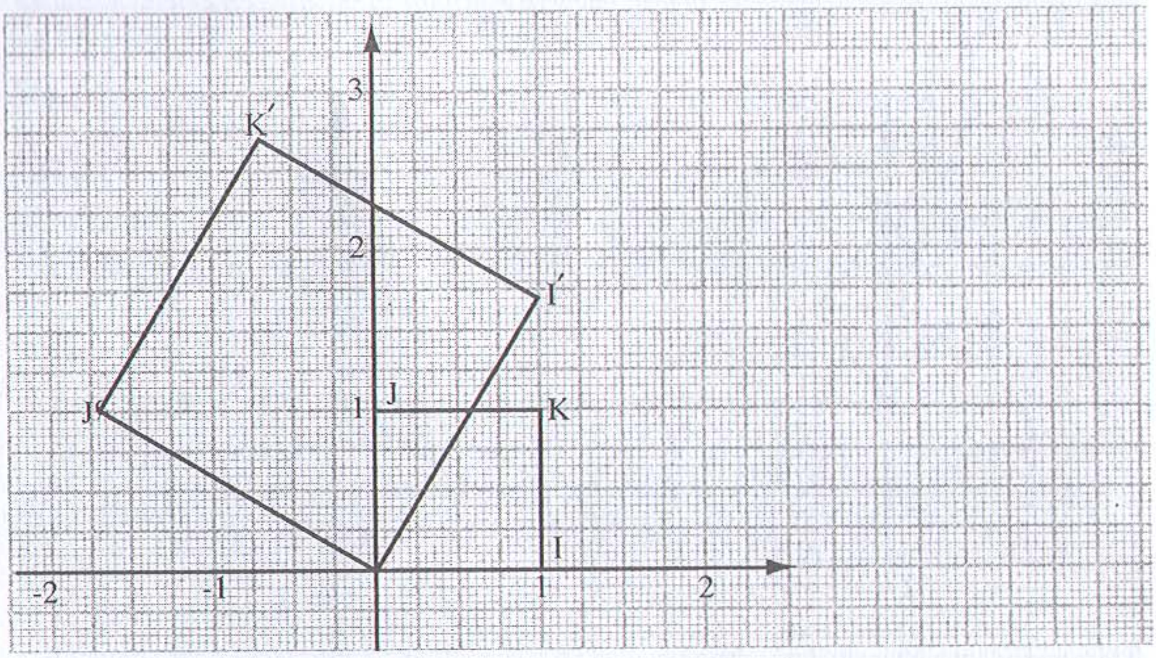
-2



3. (a) $\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$ (f) $\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$

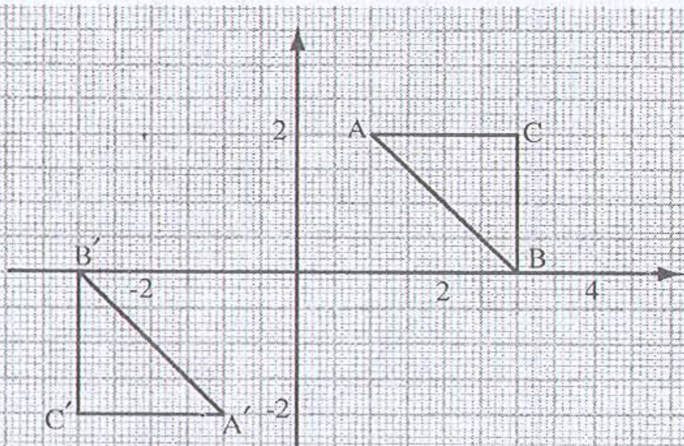
4.



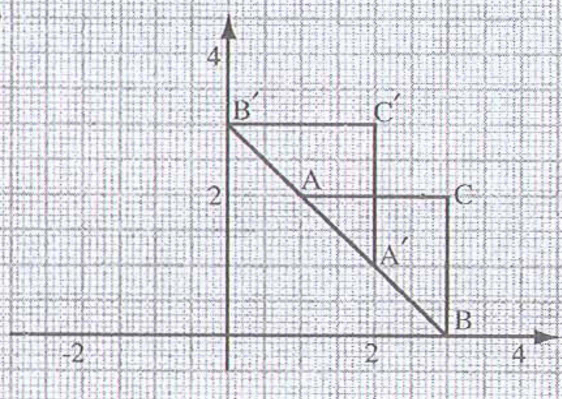
5.

Exercise 1.4

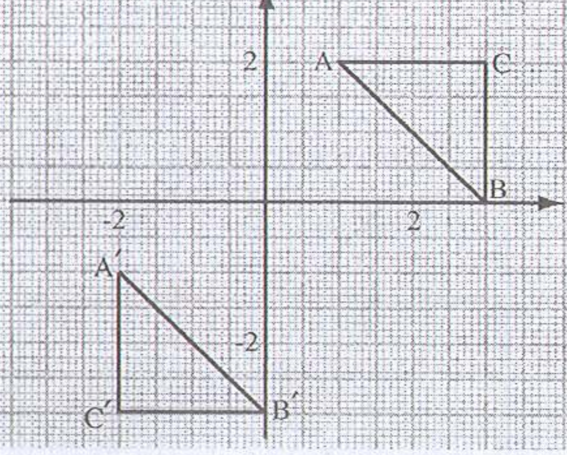
1. (a)

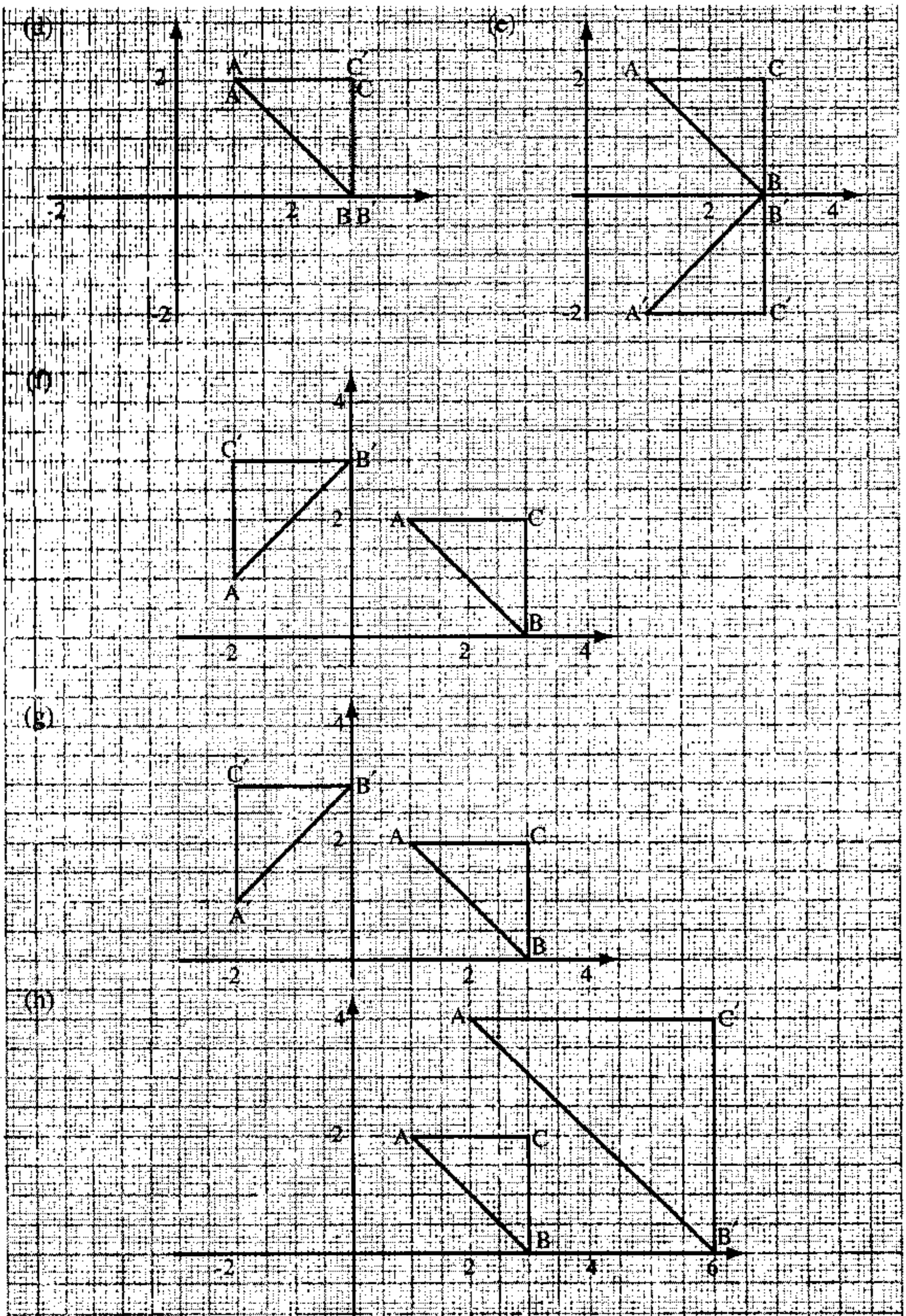


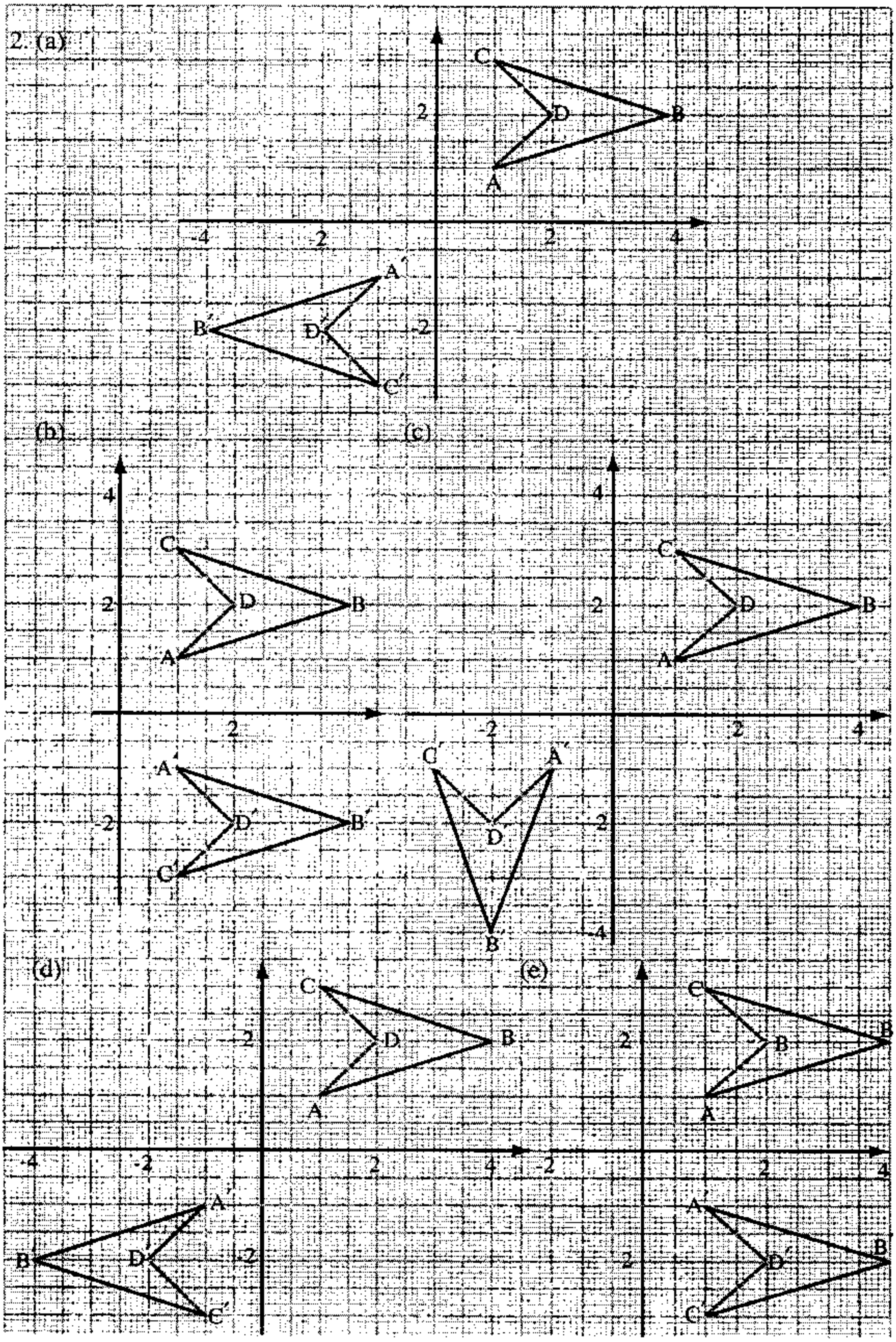
(b)

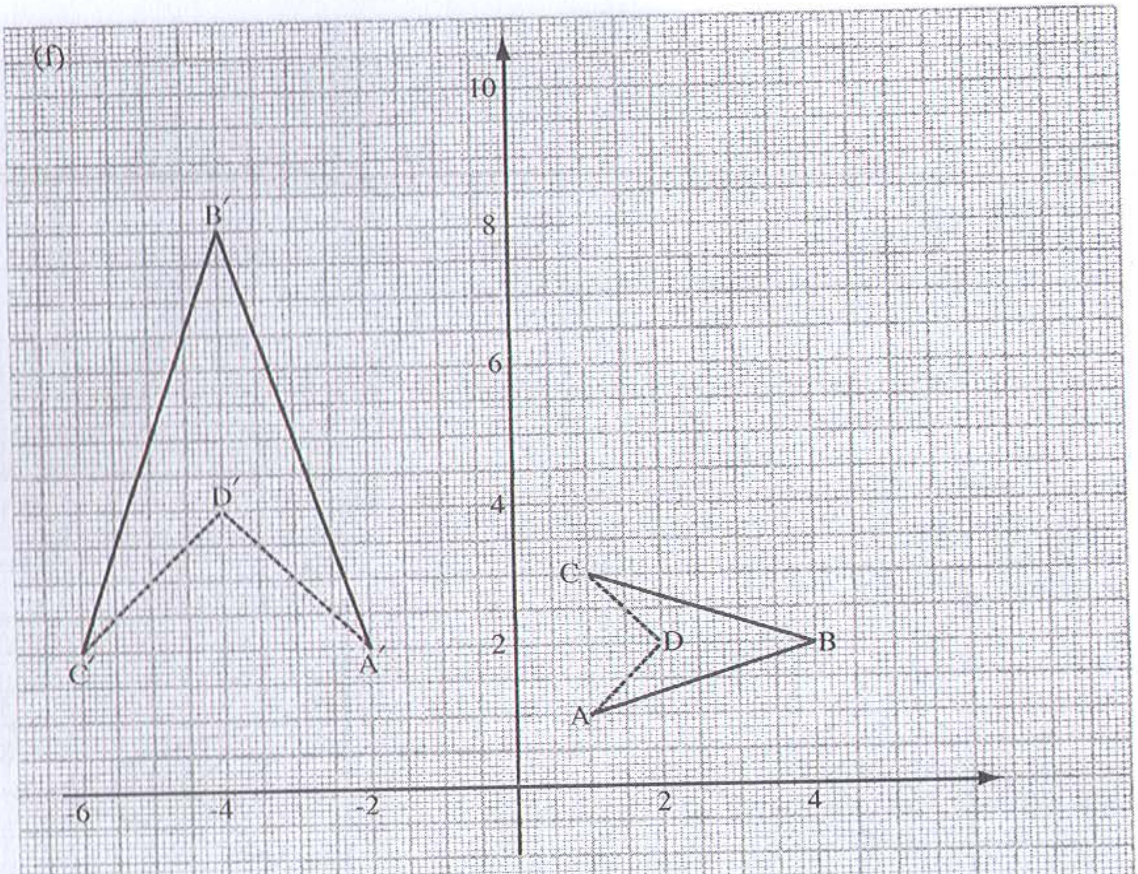


(c)




(c)
(f)
(g)
(h)
A
B
C
A′
B′
C′
-2
2
4
6


2. (a)
(b)
(c)
(d)
(e)
A
B
C
D
A′
B′
C′
D′
2
4
-2
-4



(f)
10
8
6
4
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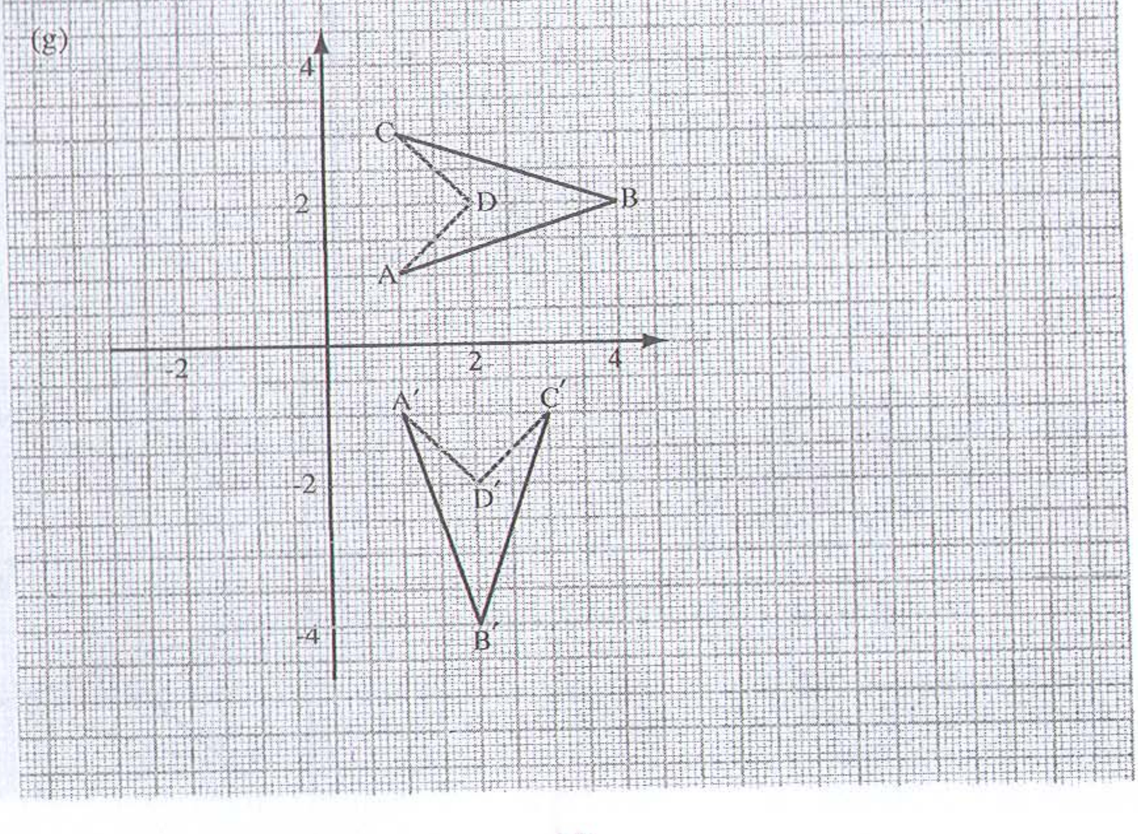
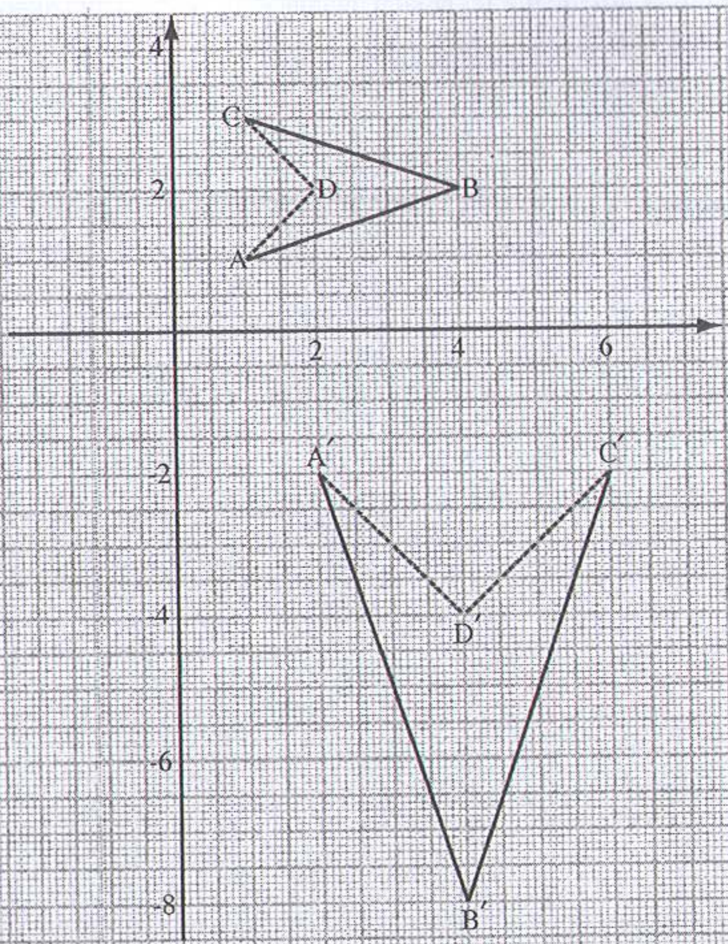
-6
-4
-2
2
4
B′
D′
C′
A′
C
D
B
A

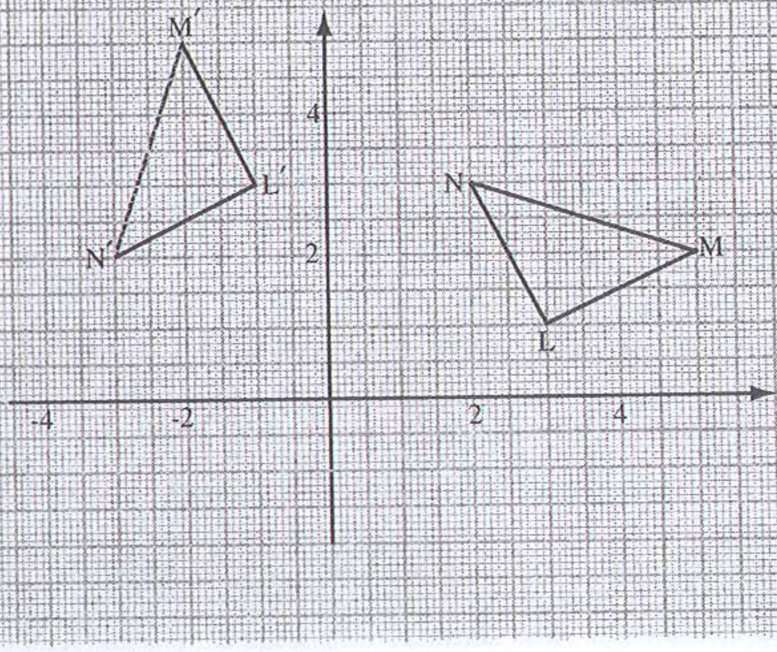


(g)
4
2
-2
-4
-2
2
4
C
D
B
A
A′
C′
D′
B′

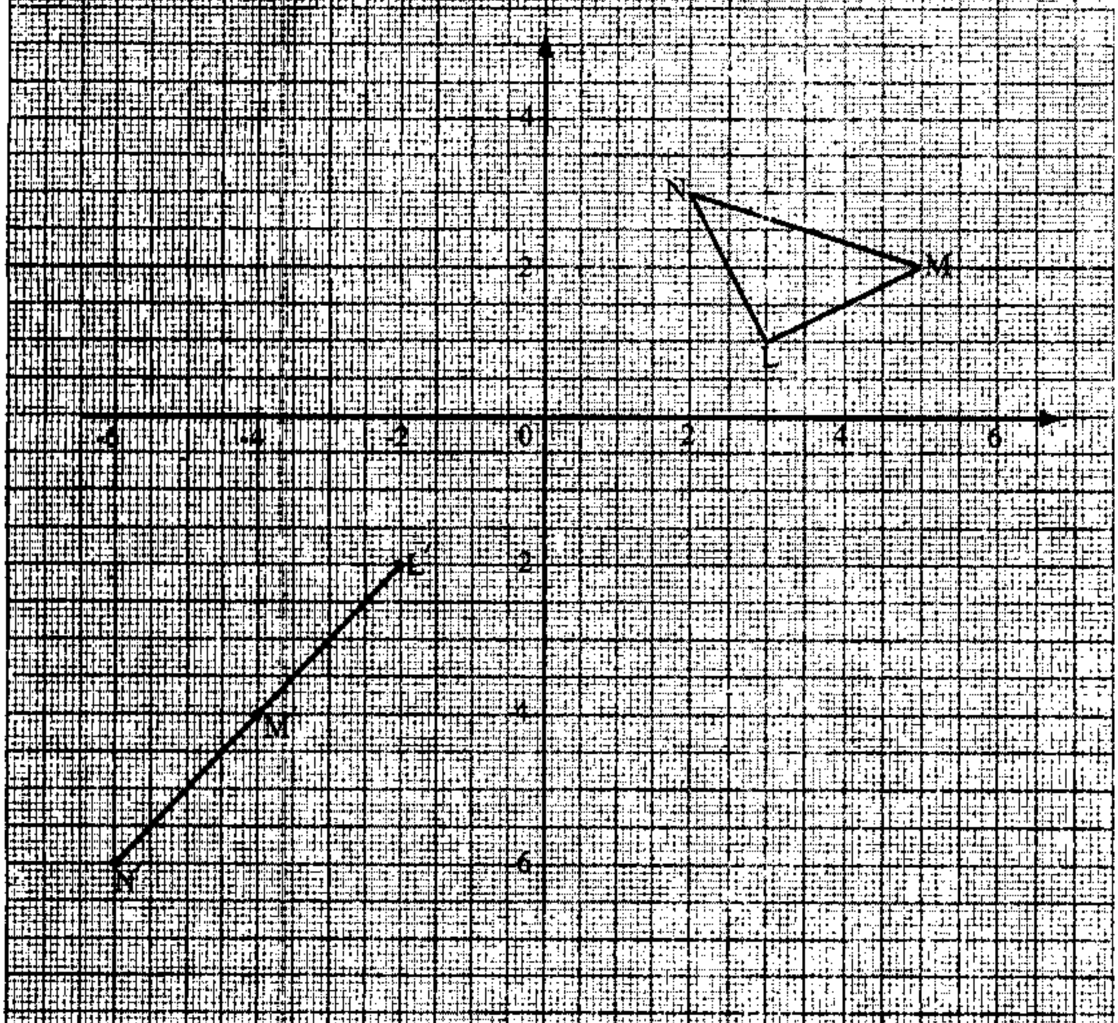
(h)



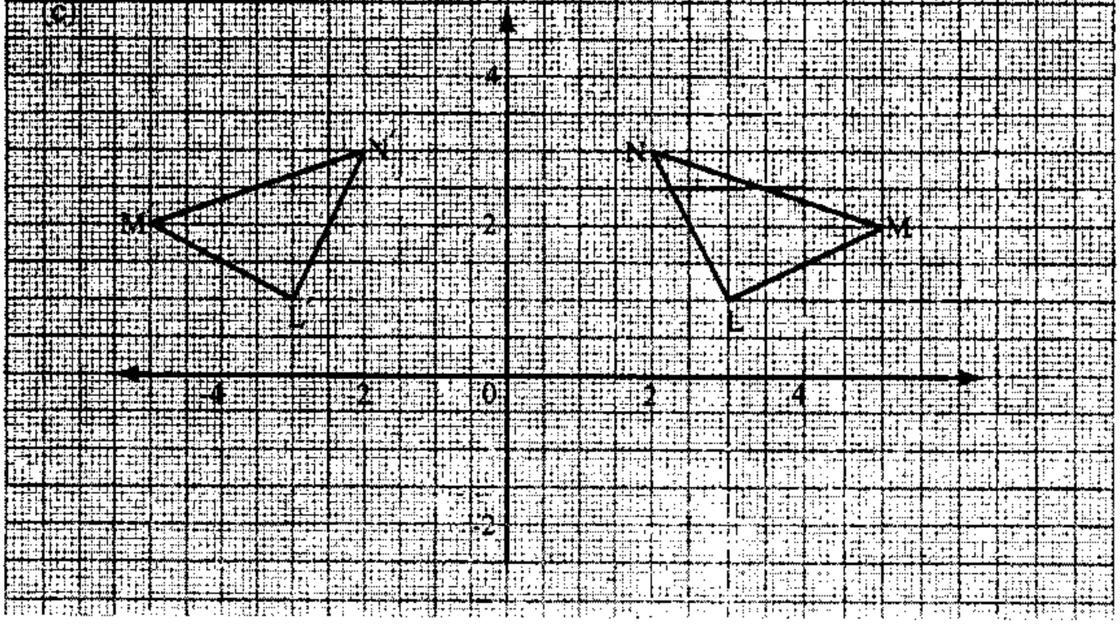
3. (a)



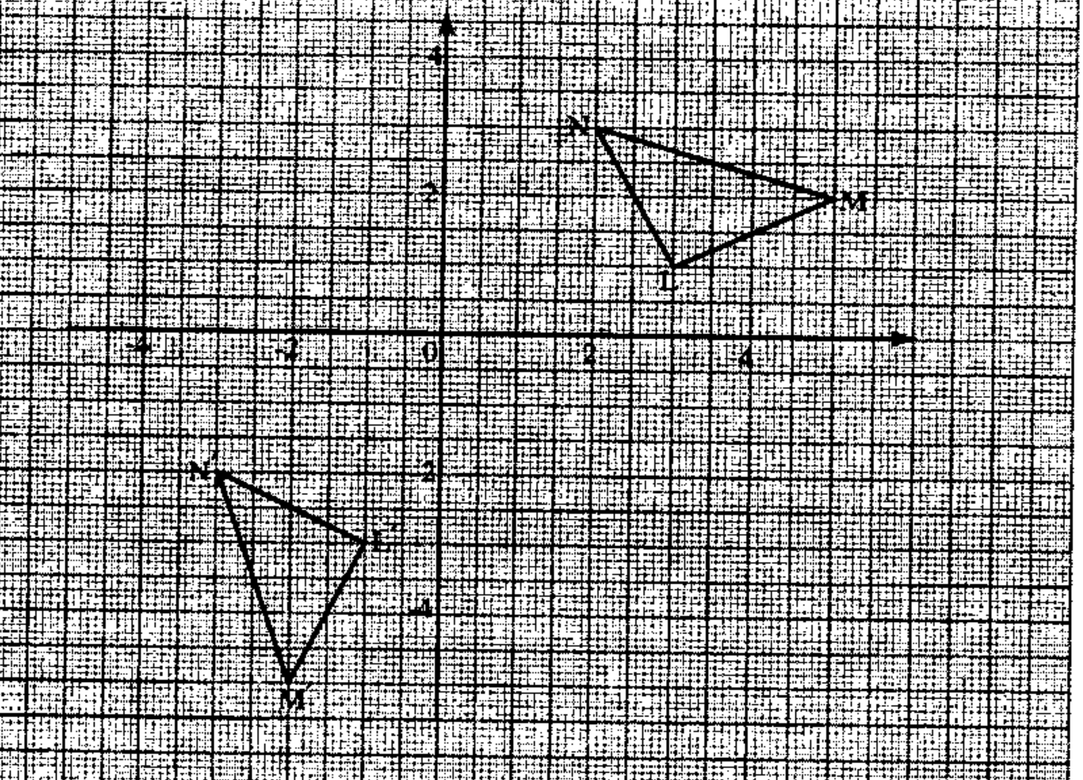
(b)



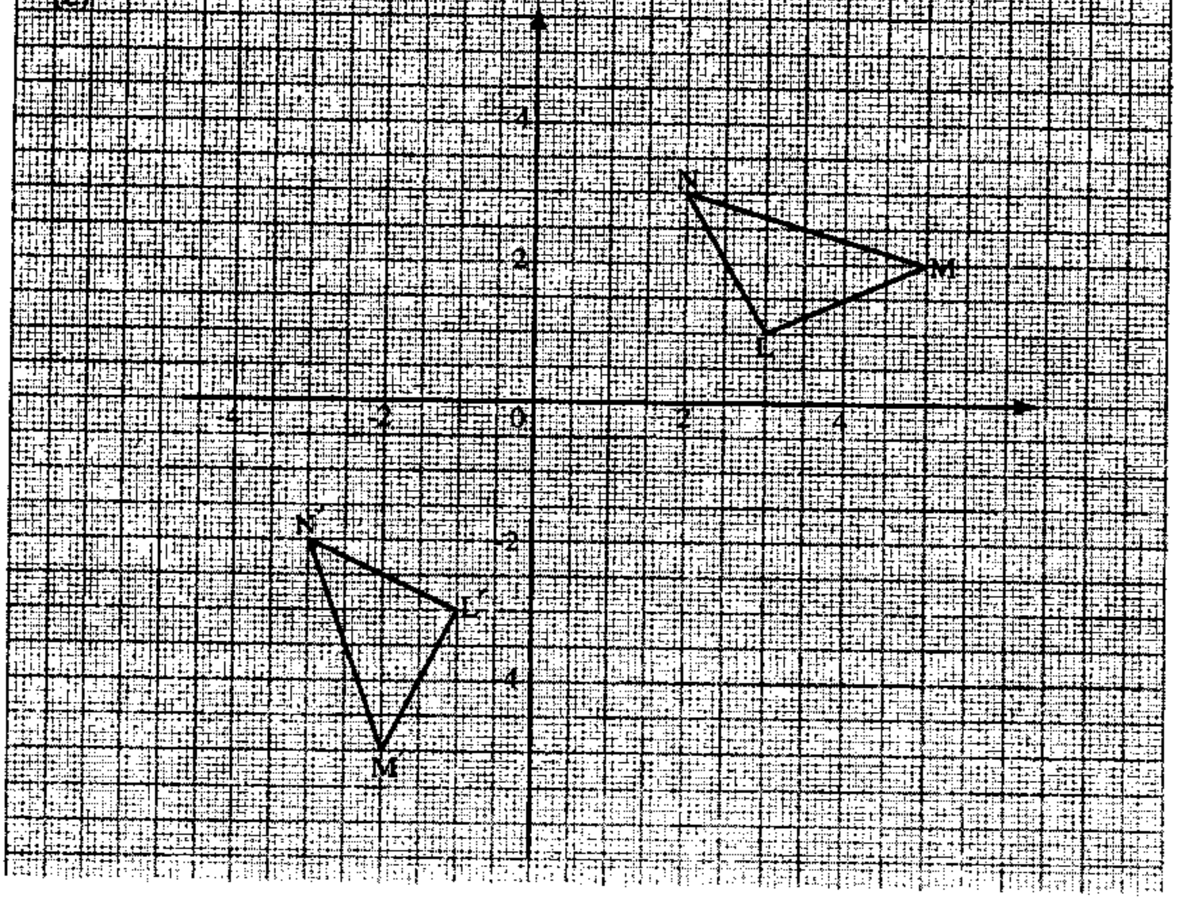
(c)

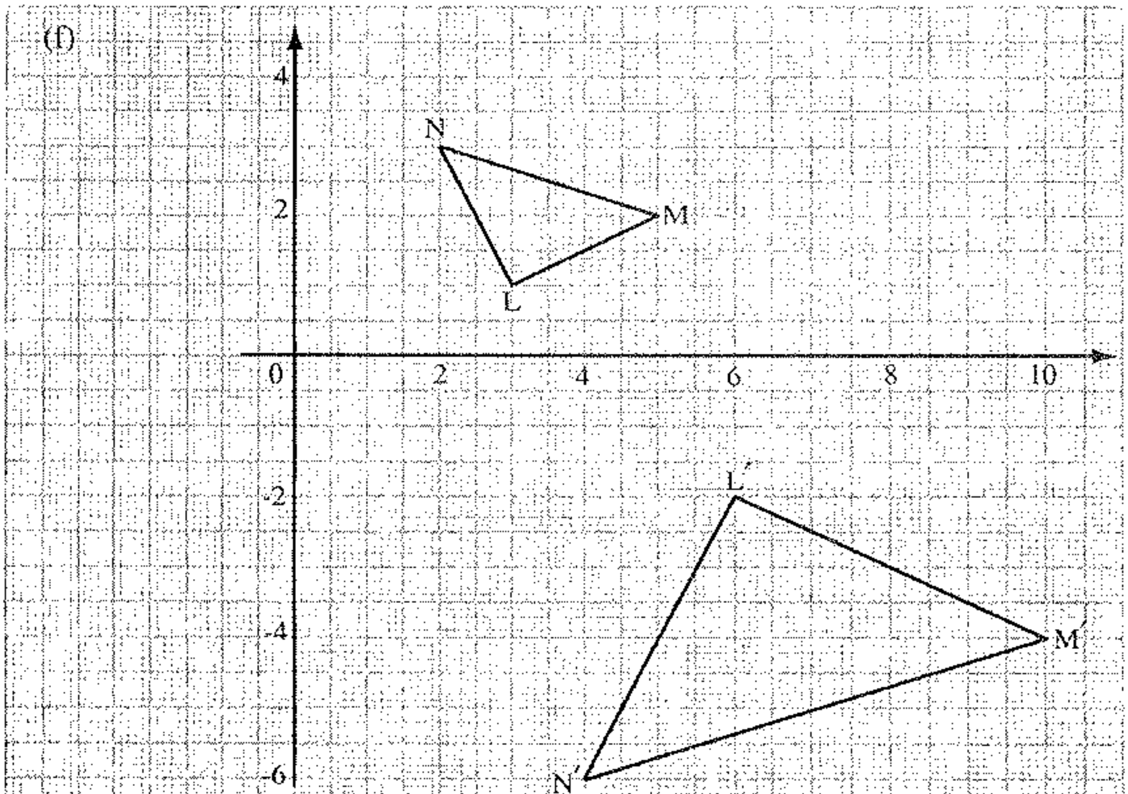


(d)



(e)




(f)
N
M
L
L′
M′
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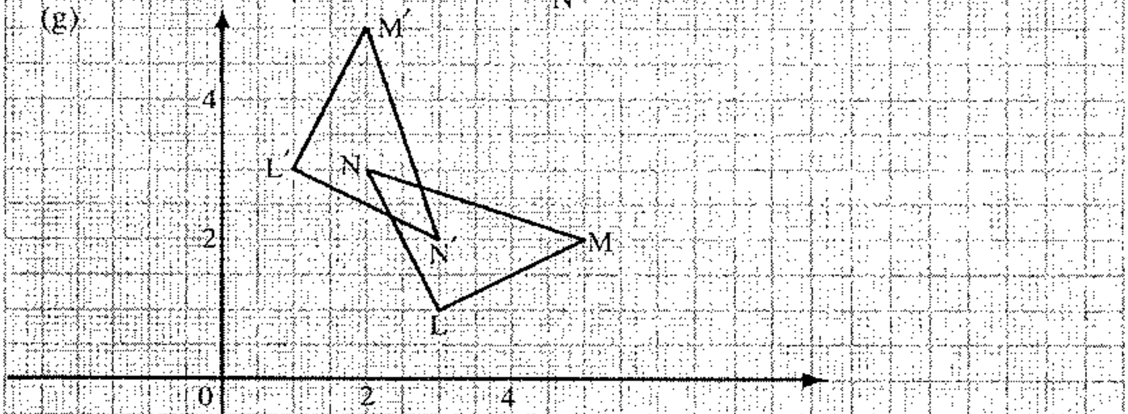
6
8
10
4
2
-2
-4
-6


(g)
M′
L′
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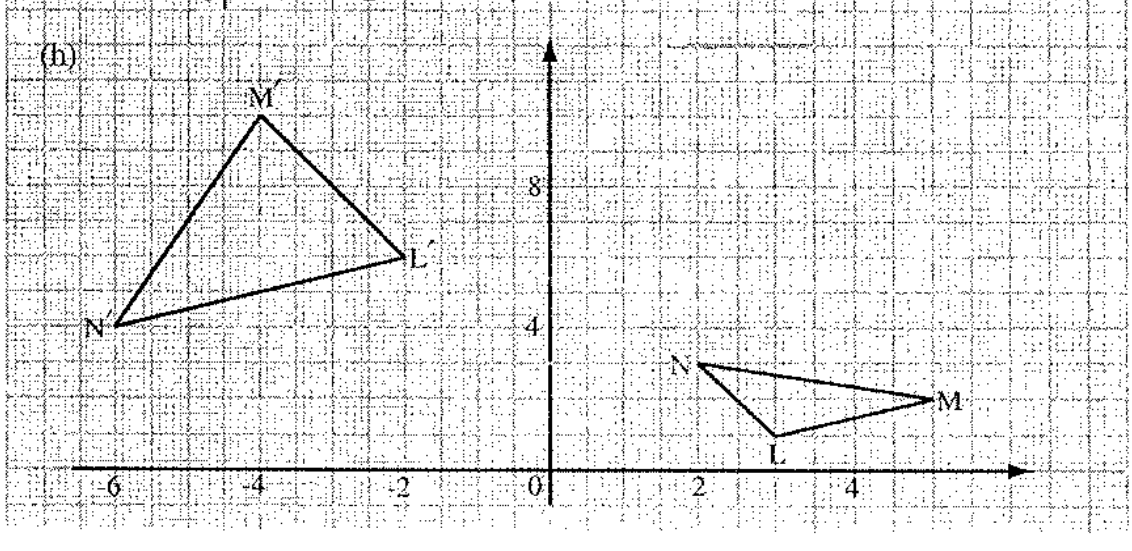
N′
M
L
4
2
0
2
4


(h)
M′
L′
N′
N
M
L
8
4
-6
-4
-2
0
2
4

4. Reflection in y-axis or line x = 0

	I	X	Y	V	U	Q	H	R
I	I	X	Y	V	U	Q	H	R
X	X	I	H	R	Q	U	Y	V
Y	Y	H	I	Q	R	V	X	U
U	U	R	Q	H	I	Y	V	X
V	V	Q	R	I	H	X	U	Y
Q	Q	V	U	Y	X	H	R	I
H	H	Y	X	U	V	R	I	Q
R	R	U	V	X	Y	I	Q	H

Exercise 1.5

1. (a) ' Subtract 5'
   (b) ' Add 4'
   (c) ' Divide by 10'
   (d) ' Multiply by 3'
   (e) 'Add 4, then divide by 10'
   (f) ' Remove socks'
   (g) ' Remove shoes, then remove socks'
   (h) ' Open the door, then get out of the room'
2. (a) Rotation about (3, 5) through -80°.
   (b) Reflection in y + x = 0
   (c) A translation, displacement vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$
   (d) Enlargement, centre (10, 10), scale factor 2.
3. (a) 50 sq. units (b) 170 sq. units
   (c) 110 sq. units (d) 0 sq. units

4. $\begin{pmatrix} -4 \\ 0 \\ -14 \end{pmatrix}$

5. 108 sq. units

6. $\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$, 86.5 square units.

7. **6 square units** $\begin{pmatrix} 0 & \frac{1}{3} \\ -1 & 0 \end{pmatrix}$

8. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$; 3 sq.units.

9. $\begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

10. (a) 30 sq. units (b) 16 sq. units (c) 4 sq. units
    (d) 56 sq. units (e) 4 sq. units

11. (a) $T^{-1}Q^{-1}$ (b) $Q^{-1}T^{-1}$ (c) $E^{-1}Q^{-1}$
    (d) $Q^{-1}E^{-1}T^{-1}$ (e) $E^{-1}T^{-1}Q^{-1}$

12. S (2, 5)
    (a) 48 sq. units
    (b) 12 sq. units
    (c) 12 sq. units
    (d) 12 sq. units
    (e) 48 sq. units

Exercise 1.6

1. (a) Shear, x-axis invariant, K (1, 1) $\longrightarrow$ K′(3, 1); $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

   (b) Shear, x-axis invariant K (1, 1) $\longrightarrow$ K′(-1, 1); $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

   (c) Negative quarter turn about (0, 0) followed by a shear, y-axis invariant (1, 0) $\longrightarrow$ (1, 2); $\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$

(d) Positive quarter turn about (0, 0) followed by a shear, y-axis invariant (-1, 1) $\longrightarrow$ ($^-$1,3); $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

(e) Positive quarter turn about (0, 0) followed by a shear, x-axis invariant (0, 1) $\longrightarrow$ (2, 1); $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

(f) Stretch, x-axis invariant, scale factor 3; $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(g) Stretch y-axis invariant or x = 0 scale factor 4; $\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$

(h) Identity transformation $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

3. (a) Non-isometric
   (b) Isometric
   (c) Isometric
   (d) Non-isometric
   (e) Non-isometric
4. K = 1 and K = -1
6. 0′ (0, 0), A′ (0, -1), B′(16, -1) C′ (16, 0)
7. Stretch, scale factor 2, x-axis invariant $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$
8. Shear, line y = 2 invariant, (-3, 5) $\longrightarrow$ (0, 5)

Chapter Two

STATISTICS II

The learner has met statistics in Book Two. In this topic, more statistical measures are dealt with.

Objectives

By the end of the topic, the learner should be able to:

(i) state the measures of central tendency.
(ii) calculate the mean using assumed mean.
(iii) make cumulative frequency table.
(iv) estimate the median and the quartiles by calculation and ogive.
(v) define and calculate the measures of dispersion, i.e., range, quartiles, interquartile range, quartile deviation, variance and standard deviation.
(vi) interpret measures of dispersion.

Time: Twenty seven lessons.

Teaching/Learning Activities

Measures of Central Tendency

- The teacher should review the measures of central tendency.
- The learner be guided on how to calculate the mean using an assumed mean.
- The teacher should lead the learner through examples 1, 2, 3 and 4.
- The learner to do exercise 2.1.
- The learner should be guided to make a cumulative frequency table.
- The teacher should discuss quartiles, deciles and percentiles, as in the students' book.
- The learner should be led through examples 5 and 6.
- The teacher should guide the learner to draw the ogive and use it to estimate the quartiles, deciles and percentiles, as in the students' book.
- The teacher should take the learner through examples 7 and 8.

- The learner to do exercise 2.2.

Measures of Dispersion

- The teacher should discuss range, interquartile range and quartile deviation.
- The learner should be guided on how to find variance and standard deviation.
- The teacher should lead the learner through examples 9 and 10.
- The learner to do exercise 2.3
- The teacher should discuss other ways of finding the standard deviations as in the students' book.
- The learner should be led through example 11.
- The learner to do exercise 2.4.

Project

- The learner should be guided in collecting data from surroundings, such as the masses of students in the class or heights of the trees. Measures of central tendency and measures of dispersion for these should then be calculated.

Answers

Exercise 2.1

1. (a) 50 (b) 41.9
2. (a) 0.7183 (b) 0.0297 (c) 435
3. 1260.2
4. 65.73 kg
5. Mode = 22, mean = 22.15, median = 22
6. Mode = 42, mean = 44.1, median = 44
7. 1.73
8. 2.934
9.

Class	*x*	*1 000x*	*t*	*f*	*ft*
0.005 - 0.006	0.0055	5.5	0	42	0
0.007 - 0.008	0.0075	7.5	2.0	38	76
0.009 - 0.010	0.0095	9.5	4.0	8	32
				$\sum f = 100$	$\sum ft = 74$

Mean percentage content = 0.00624

10. Median = 28 Mean = 27.2

11. Median = 6.2 Mean = 6.11

12. (a) 56.88 (b) 57.08

Exercise 2.2

1. (a) Mode = 2 and 7 (b) Mode = 1 357
   Median = 7 Median = 1 170
   $Q_1 = 4$ $Q_1 = 1003.5$
   $Q_3 = 11$ $Q_3 = 1\ 313$

2 (a) Mean = 39.2
  (b) Median = 38.4
  $Q_1 = 37.65$
  $Q_3 = 41$

3. (a) Modal class = 601 – 650 (b) (i) 615.34 (ii) $Q_1 = 556.06$
   $Q_3 = 756.75$

4. Median = 54.5 Mean time = 54.9

5. (a) 3 201 – 3 700
   (b) $Q_1 = 3033.8$, Q_2 = Median = 3559.9, $Q_3 = 4200.5$

6. (a) Median = 45 (b) 2^{nd} decile = 43.11, 6^{th} decile = 45.6
   (c) 29 (d) 29

7. (a) 55 (b) 1.6 kg – 23 kg (c) 50 (d) 17.3%

8. (a) Median = 34.95 (b) 4^{th} decile = 31.3
   $Q_1 = 21.5$ 6^{th} decile = 38.6
   $Q_3 = 52 .8$
   (c) 30^{th} percentile = 25.5
   70^{th} percentile = 47

9 (a) Median = 26.5
  (b) $Q_1 = 17.2$ kg, $Q_2 = 36$ kg
  (c) 5^{th} decile = median = 26.5
  7^{th} decile = 33 kg

10. (a) Mean = 33.18
    (b) (i) Median = 31.4, $Q_1 = 27$, $Q_3 = 38.6$
    (ii) 60 %
    (iii) sh. 27 – sh. 38.60
    (iv) 20^{th} percentile = 26, 80^{th} per centile = 41

11. (a) 2 (b) 2 (c) 1 – 5

Exercise 2.3

1. (a) Range = 34, Q.d = 3.75 (b) Range = 28 Q. d = 3.5
2. Mean = 4.9, standard deviation = 2.6
3. Mean = 10, standard deviation = 5.37
4. Mean = 5, standard deviation = 2.8
5. Mean = 14, standard deviation = 9.15
6. Mean = 15, standard deviation = 7.65
7. Mean = 19.63, standard deviation = 4.17
8. 11.35

Exercise 2.4

1. (a) Mean = 147.6, standard deviation = 25.65
   (b) Mean = 200, standard deviation = 178.4
2. 8.5
3. (a) Maths = 53.45 (b) Maths = 16.5
   Kiswahili = 59.7 Kiswahili = 22
4. Mean = 3 934, standard deviation = 1612.3
5. Mean = 50.3, standard deviation = 4.28
6. Mean = 43.45, standard deviation = 11.8
7. Mean = 51.9, standard deviation = 21.24
8. Mean = 220, standard deviation = 4.4
9. (a) 5589.4 (b) 1 291 (c) 1 925
10. 1.7
11. 8.12
12 Mathematics; the lower the standard deviation, the better the performance.

LOCI

The learner is conversant with geometrical constructions and properties of a circle. This knowledge will be useful as a foundation for this topic.

Objectives

By the end of the topic, the learner should be able to:

(i) define locus.

(ii) describe common types of loci.

(iii) construct:

- loci involving inequalities.
- loci involving chords.
- loci involving points under given conditions.

Time: Twenty one lessons,

Teaching/Learning Activities

Introduction to Loci

- The teacher should define the term 'locus' using practical examples, as in the students' book.
- The learner should be guided through actvities such as those in the students' book.
- The learner to do exercise 3.1.

Common Types of Loci

- The teacher should discuss the perpendiculor bisector locus, as in the students' book.
- The learner should be guided in determination of the locus of a point which moves equidistant from a given line, as in the students' book.
- The learner should be led in determining the locus of a point which is equidistant from another fixed point, as in the students' book.
- The learner should be guided to find the locus of a point which is equidistant from two intersecting lines.
- The teacher should guide the learner through example 1.

- The teacher should discuss constant angle loci, as in the students' book.
- The learner should be led through example 2.
- The learner to do exercise 3.2.

Intersecting Loci

- The teacher should discuss intersection of loci.
- The learner should be taken through examples 3 and 4.
- The learner should be guided in locating intersecting loci in a triangle, as in the students' book.
- The learner to do exercise 3.3.

Loci of Inequalities

- The teacher should introduce the learner to the loci of points satisfying one or more inequalities.

Loci Involving Chords

- The teacher should guide the learner in constructing loci involving chords, as in the students' book.
- The learner to do exercise 3.5.

Additional Hints

- Emphasis should be put on the language of loci. Different ways of describing the same loci should explored.
- The teacher should make use of available resources as much as possible.

Evaluation

A written test on loci should be given to the learner.

Mixed Exercise 1

1. (a) A shear with x-axis invariant, (0, 1) is mapped onto (3, 1).
   (b) Rotation through + 90° about the origin.
   (c) Enlargement, centre (0, 0), scale factor 2.
   (d) Reflection in the x-axis.
2 $x = 1, y = -1$
3. (a) $a = 1, b = 0,\ c = 0$ and $d = 1$
   (b) Reflection in the y-axis.
4. $x = 5$

5. (a) Perpendicular bisector of QR.
   (b) Sphere radius x cm.
   (c) A circle of diameter XY.
6. Mean = 3.267, median = 3, mode = 3
7. Modal class = 39 – 45
   Mean = 40.95
8. (a) $P'(12, 12), Q'(8, 4), R'(18, 12)$
   (b) $P'(13, 1), Q'(5, -1), R'(14, -1)$
   (c) $P'(2, -18), Q'(-2, -6)\ R'(-2, -18)$
   (d) $P'\left(\frac{3}{2}, 3\right), Q'\left(\frac{5}{6}, \frac{5}{3}\right), R'(2, 4)$
9. (a) $P'(0, 0), Q'(8, 0), R'(0, 2)$
   (b) $P''(0, 0), Q''(8, 0), R''(0, 4)$
   (c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ Enlargement, centre (0, 0), scale factor 2
10. The locus of point P is two parallel lines on either side of QR and a distance of 5 cm from it.
11. (a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
12. (a) $P'(0, 0), Q'(24, 6). R''4, 4)$
    (b) 36 sq. units.
13. A circle with diameter PQ = 6 cm.
14. (a) $x = 1$ (b) $y = 0$
15. $\begin{pmatrix} 18 & 32 & 5 \\ 42 & 48 & 131 \\ -2 & 4 & -25 \end{pmatrix}$
16. 16
18. (a) 60
    (b) 48.18
    (c) Bimodal 30 and 50, median = 46.5
19. 6.5 cm
21. (a) 56.5 (b) 26 (c) 76 %

22. $y = x + 4$ and $y + x = 4$

23. (a) A circle of diameter AB

(b) Two major segments with a common chord AB.

24. (a) 5.6

(b) Median = 4.79, Q_1 =2.4, Q_3 = 8.3

(c) 3.75

25. (a) 62.02 (b) 63.1 (c) 10.1

26. (a) 172.9 (b) 160 (c) 12.65

27. (a) T is 3.8 cm on either side of line PQ and on the angle bisector of PRQ.

28. (a) 19.5 (c) 19.25 (c) 70 %

29. (a) 5.55 (b) 0.52

30. (a) 84.05 km (b) 104.44°

31 (a) A′(4, 6), B′(12, 18), C′(18, 36), D′(10, 24); Area =72 sq. units

(b) $\begin{pmatrix} \frac{1}{2} & \frac{-1}{6} \\ 0 & \frac{1}{6} \end{pmatrix}$

32.

Class	*Midpoint (x)*	*Frequency (f)*	*fx*
50 – 54	52	5	**260**
55 – 59	57	3	**171**
60 – 64	62	**7**	434
65 – 69	**67**	**5**	335
70 – 74	72	5	**360**
75 – 79	**77**	**3**	231
80 – 84	82	4	**328**
		$\sum f = 32$	$\sum fx = 2\,119$

Mean = 66.2; standard deviation = 9.53

33. Circle with same centre as the given circle and radius 4 cm.

Chapter Four

TRIGONOMETRY III

The learner has been exposed to trigonometry in Books Two and Three. In this topic, the learner will be taken through solutions of trigonometric equations and graphs.

Objectives

By the end of the topic, the learner should be able to:

(i) recall and define trigonometric ratios.

(ii) derive the trigonometric identity $\sin^2 x + \cos^2 x = 1$.

(iii) draw graphs of trigonometric ratios of the form;
$y = \sin x$, $y = \cos x$, $y = \tan x$,
$y = a \sin x$, $y = a \cos x$, $y = a \tan x$,
$y = a \sin bx$, $y = a \cos bx$, $y = a \tan bx$,
$y = a \sin (bx \pm \theta)$, $y = a \cos (bx \pm \theta)$, $y = a \tan(bx \pm \theta)$

(iv) solve simple trionometric equations analytically and graphically.

(v) deduce from the graph, amplitude, period, wavelenth and phase angles.

Time: Twenty one lessons.

Teaching/Learning Activities

Trigonometric Ratios

- The teacher should review trigonometric ratios as covered in Book Two.
- The learner should be guided in deriving the trigonometric identity, as in the students' book.
- The teacher should lead the learner through examples 1, 2 and 3.
- The learner to do exercise 4.1.

Waves

- The teacher should discuss amplitude and period of a wave, as in the students' book.
- The learner to do exercise 4.2.

- The teacher should guide the learner through a discussion of some transformations of waves, as in the students' book.
- The learner should be guided through example 4.
- The teacher should discuss translation in relation to waves as in the students' book.
- The learner to do exercise 4.3.

Trigonometric Equations

- The teacher should lead the learner to solve trigonometric equations analytically, as in the students' book.
- The learner should be guided through example 4.
- The teacher should guide the learner in obtaining graphically solutions to trigonometric equations, as in example 5.
- The learner to do exercise 4.4.

Additional Hints

- The teacher should lay great emphasis on the need to exhaust all the values in the indicated range when solving trigonometric equations.
- The learner should be encouraged to make use of a calculator in computations in the topic.

Evaluation

- The teacher should give a written test on trigonometry.

Further Questions

1. Solve the following trigonometric equations, giving your answers in degrees for $0° \leq x \leq 360°$.
   (a) $\cos x - \sin x = 1$
   (b) $\sin 3x + \cos 3x = 1$
2. Describe fully a transformation that maps $y = \cos 2x$ onto:
   (a) $y = 1 + \cos 2x$
   (b) $y = \cos 2x - \frac{1}{2}$

Answers

Exercise 4.1

1. (a) $\frac{\sqrt{15}}{4}$ (b) 15 (c) $1\frac{1}{15}$

2. 1

3. Prove that $\tan\theta + \frac{1}{\tan\theta} = \frac{1}{\sin\theta\cos\theta}$

$$\tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

4. Prove $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$.

$\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 1(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

5. Prove that $\sin\theta\cos^2\theta = \sin\theta - \sin^3\theta$

$\sin\theta\cos^2\theta = \sin\theta(1 - \sin^2\theta)$

$= \sin\theta - \sin^3\theta$

6. Prove that $\frac{\cos\theta - \tan\theta}{\sin\theta} - \cos^2\theta = \sin^2\theta$

$$\frac{\cos\theta\tan\theta}{\sin\theta} - \cos^2\theta = \frac{\cos\theta}{\sin\theta}\frac{\sin\theta}{\cos\theta} - \cos^2\theta$$

$$= 1 - \cos^2\theta$$

$$= \sin^2\theta$$

7. Prove that $\dfrac{\sin^3\theta}{\cos\theta} = \tan\theta - \cos^2\theta\tan\theta$

$$\frac{\sin^3\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}\sin^2\theta$$

$$= \tan\theta\,(1 - \cos^2\theta)$$

$$= \tan\theta - \cos^2\theta\,\tan\theta$$

8. Prove that $\dfrac{1}{\cos^2\theta} - \cos^2\theta - \dfrac{\sin^2\theta}{\cos^2\theta} = \sin^2\theta.$

$$\frac{1}{\cos^2\theta} - \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} - \frac{(\cos^4\theta + \sin^2\theta)}{\cos^2\theta}$$

$$= \frac{1 - \sin^2\theta - \cos^4\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta - \cos^4\theta}{\cos^2\theta}$$

$$= (1 - \cos^2\theta)$$

$$= \sin^2\theta$$

9. Prove that $\dfrac{\tan^2\theta + 1}{\tan^2\theta} = \dfrac{1}{1 - \cos^2\theta}$

$$\frac{\tan^2\theta + 1}{\tan^2\theta} = 1 + \frac{1}{\tan^2\theta}$$

$$= 1 + \frac{\cos^2\theta}{\sin^2\theta}$$

$$= 1 + \frac{1-\sin^2\theta}{\sin^2\theta}$$

$$= 1 + \frac{\cos^2\theta}{\text{sins}^2\theta}$$

$$= 1 + \frac{1-\sin^2\theta}{\sin^2\theta}$$

$$= 1 + \frac{1}{\sin^2\theta} - 1$$

$$= \frac{1}{1-\cos^2\theta}$$

10. If $x = 2\cos\theta$, show that $\frac{\sqrt{4-x}}{x} = \tan\theta$.

(Consider positive square root only)

$$\frac{\sqrt{4-x^2}}{x} = \frac{\sqrt{4-(2\cos\theta)^2}}{2\cos\theta}$$

$$= \frac{\sqrt{4-4\cos^2\theta}}{2\cos\theta}$$

$$= \frac{2\sqrt{1-\cos^2\theta}}{2\cos\theta}$$

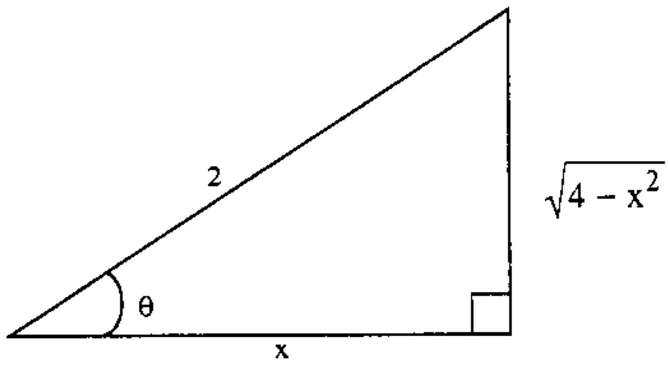
$$= \frac{\sqrt{\sin^2\theta}}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

Alternatively;

$\cos\theta = \frac{x}{2}$



$\therefore \tan\theta = \dfrac{\sqrt{4 - x^2}}{x}$

11. If y = cos θ, show that $y\sqrt{(2y^2 - 1)} = \sqrt{(2\sin^2\theta - 1)(\sin^2\theta - 1)}$

$y = \cos\theta$

Therefore; $y\sqrt{(2y^2 - 1)} = \cos\theta\sqrt{2\cos^2\theta - 1}$

$$= \cos\theta\sqrt{2(1 - \sin^2\theta) - 1}$$

$$= \cos\theta\sqrt{2 - 2\sin^2\theta - 1}$$

$$= \cos\theta\sqrt{1 - 2\sin^2\theta}$$

$$= \sqrt{\cos^2\theta(1 - 2\sin^2\theta)}$$

$$= \sqrt{(1 - \sin^2\theta)(1 - 2\sin^2\theta)}$$

$$= \sqrt{(\sin^2\theta - 1)(2\sin^2\theta - 1)}$$

12. (a) $\dfrac{23 - 10\sqrt{2}}{27 - 10\sqrt{2}}$ (b) $\dfrac{92 + 40\sqrt{2}}{129}$

Exercise 4.2

1. (a) (i) 1 (b) (i) 180°
   (ii) 2 (ii) 360°

(iii) 1 (iii) 360°
(iv) 1 (iv) 720°

2. (a) Check for accuracy of graph.
   Amplitude 1, period 360°
   (b) Check for accuracy of graph.
   Amplitude 2, period 360°
   (c) Check for accuracy of graph.
   Amplitude 3, period 360°

Exercise 4.3

1. (a) Check for accuracy of curves.
   A stretch with y-axis invariant and scale factor 3
   (b) Check for accuracy of curves.
   A stretch with x-axis invariant and scale factor 4
   (c) Check for accuracy of curves.
   A translation of vector $\binom{60°}{0}$
2. Check for accuracy of graph
   $y = 2\cos(x - \frac{\pi}{3})c$
3. Check for accuracy of graph.
   A translation of vecto $\binom{60°}{0}$
4. (a) $y = 3 \sin x$
   (b) $y = 2 \sin 3x$
   (c) $y = \sin (x \pm \frac{\pi}{3})c$
   (d) $y = 2\sin (x + \frac{\pi}{2})^c$
5. (i) Amplitude 1, $\frac{1}{4}$; $\frac{1}{4}$ of first
   Period 360°, 360°; same
   (ii) Amplitude 3, $\frac{3}{2}$; $\frac{1}{2}$ of first
   Period 360°, 720°; twice the first.
   (iii) Amplitude 1, 4; 4 times the first.
   Period 120°, 360°; thrice the first.

(iv) Amplitude I, k times the first.

Period 360°, $\left(\frac{360}{a}\right)^{\circ}, \frac{1}{a}$ times the first

6. Check for the accuracy of the graph.
   (a) (i) 0.9 (ii) −1.7
   (b) (i) 0.53 or 1.8 (ii) 0.93 or 1.4
7. Check for accuracy of graphs.
   Amplitude 1.42, pseriod 360°
8. (a) a units

   (b) $\left(\frac{360}{b}\right)^{\circ}$

   (c) c°
9. Check for accuracy of graphs.
   $y = 3\sin\left(2t + \frac{\pi}{2}\right)$; Amplitude 3, period 180°
   $y = 4\sin\left(2t - \frac{\pi}{2}\right)$; Amplitude 3, period 180°

Exercise 4.4

1. (a) $\frac{\pi^c}{6}$ (b) $\frac{\pi^c}{3}$ or $\frac{4}{3}\pi^c$

   (c) $0^c, \frac{2}{3}\pi^c, \pi^c$ or $\frac{5}{3}\pi^c$ (d) $\frac{\pi^c}{12}$ or $\frac{13}{12}\pi^c$
2. (a) 60° or 180° (b) 173°32′ or 306°28′
   (c) 27°, 153° 207° or 333°
3. (a) $\frac{\pi^c}{6}$ or $\frac{5}{6}\pi^c$ (b) $\frac{\pi^c}{6}$ or $\frac{11}{6}\pi^c$ (c) $\frac{\pi^c}{3}$ or $\frac{4}{3}\pi^c$

   (d) $\frac{\pi^c}{6}, \frac{5}{6}\pi^c, \frac{7\pi^c}{6}$ or $\frac{11\pi^c}{6}$ (e) $\frac{\pi^c}{2}$ or $\frac{3\pi^c}{2}$
4. (a) 45°, 135°, 225° and 315°

   (b) $\frac{\pi^c}{6}$ or $\frac{5\pi^c}{6}$
5. 33°46′, 53°14′, 123°46′, 146°14′, 213°46′, 236°14′, 303° 46′ or 326°14′
6. Check for accuracy of curves.
   x = 15° or 175°
7. Check for accuracy of curves.

$x = \frac{\pi^c}{5}$ or $\frac{71}{90}\pi^c$

8. Check for accuracy of graphs.
   (a) $x = 9°, 48°, 192°$ or $218°$
   (b) $x = 0°, 60°, 180°, 240°, 360°$
   (c) $3° \leq x \leq 54°$ or $189° \leq x \leq 234°$
9. Check for accuracy of graph
   $x = 27°, 39°, 123°, 195°, 219°, 318°$ or $352°$

Further Questions

1. (a) 0° and 360°
   (b) 0°, 30° and 120°

2. (a) Translation of vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

   (b) Translation of vector $\begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}$

Chapter Five

THREE-DIMENSIONAL GEOMETRY

The topic is new to the learner. The learner has however met solids, Pythagoras' theorem and trigonometry, which are pre-requisite concepts for the topic.

Objectives

By the end of the topic, the learner should be able to:

(i) state the geometric properties of common solids.

(ii) identify projection of a line onto a plane.

(iii) identify skew lines.

(iv) calculate the length between two points in three dimensional geometry.

(v) identify and calculate the angle between:

- two lines.
- a line and a plane.
- two planes.

Time: Twenty four lessons.

Teaching/ Learning Activities

Geometrical Properties of Common Solids

- The teacher should discuss geometrical properties of solids, as in the students' book.

Angle Between a Line and a Plane

- The teacher should guide the learner in identifying the projection of a line onto a plane, as in the students' book.
- The learner should be led to identify the angle between a line and a plane.
- The teacher to guide the learner through example 1.
- The teacher should guide the learner through example 2.
- The learner to do exercise 5.1.

Angle Between two Planes

- The teacher should guide the learner in identifying the angle between two planes, as in the students' book.
- The learner should be guided on how to find the angle between two planes, as in examples 3 and 4.

Skew Lines

- The teacher should lead the learner in identifying skew lines, as in the students' book.
- The learner should be led in finding the angle between skew lines, as in the students' book.
- The learner to do exercise 5.2.

Additional Hints

A practical approach should be used in this topic as much as possible.

Evaluation

The teacher should give a written test on this topic.

Further Questions

1. A regular tetrahedron of side 6 cm rests on one of its faces. Find
   (a) the altitude of the tetrahedron.
   (b) the angle between any of the slanting faces and the base.

Answers

Exercise 5.1

1. (a) (i) PR (ii) PR (iii) PR (iv) AC (v) DR
   (vi) ON (vii) N
   (b) (i) $38°40'$ (ii) $0°$ (iii) $38°40'$ (iv) $38°40'$ (v) $38°40'$
   (vi) $32°3'$ (vii) $90°$

2. (a) (i) 4 cm (ii) 4.717 cm (iii) 6.403 cm (iv) 6.403 cm
   (b) (i) $58°$ (ii) $38°40'$ (iii) $45°$ (iv) $58°$
   (v) $32°3'$

3. $90°$, $73°44'$, $58°$, $64°6'$

4. (a) $53°\ 8'$ (b) 10 cm

5. (a) 14.31 cm (b) $61°56'$ (c) $28°4'$ (d) 511. 9 cm^3

6. (a) 13 cm (b) 4.615 cm (c) $28°18'$

7.	(a)	10 cm	(b)	11.18 cm	(c)	26°34′	
8.	(a)	5.59 cm,	3.905 cm and 4.717 cm				(b) 30°
9.	(a)	6.359 cm	(b)	13 cm	(c)	29.28′	
10.	(a)	50 cm	(b)	51.78 cm	(c)	52° 19′	
11.	(a)	16.16 cm ,	17 cm		(b)	61° 56′	
12.	(a)	15.36 cm	(b)	65°25′	(c)	$5\sqrt{2}$	(d) 15. 695 cm
	(e)	17 cm	(f)	22°59′	(g)	923.2 cm^3	

Exercise 5.2

1. (a) Any two from PQ, SR, TU, WV or PS, QR, VU, WT or PW, QV, RU, ST.
   (b) PW and PQ or PS, PQ and QR or QV, PS and SR or ST, QR and RS or RU, PW and WT or WV, WV and QV or VU, VU and UR or UT and WT and TU or TS.
   (c) PWVQ and SRUT, QRUV and PSTW or PQRS and WVUT.
   (d) PQVW and PSRQ or QRUV or WTUV or PSTW,
   PSTW and SRUT or PSRQ or WTUV
   QRUV and WTUV or TURS or PSQR.
   SRUT and WTUV or SRQP, and so on.
   (e) PSRQ, PQVW and QRUV, QRUV, PQRS and SRUT, WVUT, QRUV and SRUT, and so on.
   (f) PW and RS, PQ and ST, QV and SR, QR and ST, and so on.
   (g) PW and SR, PW and SQ, QV and SU, and so on.
   (h) PQVW, PSRQ and PQUT, PQVW, PSTW and PRUW, PQVW, QRUV and QVTS, and so on.
   (i) WQ, QU and WU, PT, TR and PR, SQ, SU and QU, and so on.
   (j) WQU, QVW, QVU and WVU, VUT, VUR, RUT and TVR, WTV, SUT, STW and SWU, and so on.

2.	(a)	(i)	TO	(ii)	BC
	(b)	(i)	45°14′or 44°46′	(ii)	23°s25′
3.	(a)	(i)	KL	(ii)	TW
	(b)	(i)	90°	(ii)	90°

(c) (i) 45° (ii) 60°

4. 53° 8′, $\frac{4}{3}$

5. (a) 63°26′ (b) 26°34′, 0.5

6. (a) (i) 90° (ii) 63° 26′
(b) 68°12′

7. (a) 36°52′ (b) 22°35′

8. (a) 30°58′ (b) 46°41′

9. 9 (a) 56°19′ (b) 90°

10. (a) 10 cm (ii) 7. 211 cm
(b) (i) 21°48′ (ii) 47°58′ (iii) 33°51′

11. (a) 36° (b) 72° (c) 36° (d) 72° or 108°

12. (a) 30 cm
(b) (i) 15 cm (ii) 16 cm
(c) (i) 38°40′ (ii) 45°

Further Questions

1. (a) $2\sqrt{6}$ or 4.899 cm (b) 70°32′

Chapter Six

LONGITUDES AND LATITUDES

This topic is new to the learner. However, the learner has dealt with trigonometric ratios, length of an arc and cartesian co-ordinates, which are pre-requisite knowledge for the topic. The learner will be exposed to problems involving longitudes and latitudes.

Objectives

By the end of the topic, the learner should be able to:

(i) define the great and small circles in relation to a sphere (including the earth).
(ii) establish the relationship between the radii of small and great circles.
(iii) locate a place on the earth's surface in terms of latitude and longitude.
(iv) calculate the distance between two points along the great circles and small circles (longitude and latitude) in nautical miles (nm) and kilometres (km).
(v) calculate time in relation to longitudes.
(vi) calculate speed in knots and kilometres per hour.

Time: Twenty one lessons.

Teaching/Learning Activities

Great and Small Circles

- The teacher should discuss the great and small circles, as in the students' book.
- The teacher should introduce latitudes and longitudes, as discussed in the students' book.

Position of a Place on the Earth's Surface

- The learner should be guided in finding the position of a place on the earth's surface, as in the students' book.
- The teacher should lead the learner through example 1.
- The learner to do exercise 6.1.

Distances on the Surface of the Earth

- The learner should be introduced to distance along great circles, as in the students' book.
- The teacher should guide the learner through examples 2 and 3.
- The learner should be led in establishing the relationship between the radii of small circle and great circles as in the students' book.
- The learner should be led through example 4.
- The teacher should discuss distances on small circles in nautical miles.
- The learner should be taken through examples 5 and 6.
- The teacher should guide the learner in calculation of shortest distances between two points on the earth's surface, as in example 7.
- The learner to do exercise 6.2.

Longitude and Time

- The teacher should discuss the relationship between time and longitude, as in the students' book.
- The learner should be led through examples 8, 9 and 10.

Speed

- The learner should be introduced to speed in knots, as in the students' book.
- The teacher should lead the learner through example 11.
- The learner to do exercise 6.3.

Evaluation

- The teacher should give a written test on longitudes and latitudes.

Answers

Exercise 6.1

1. A (80°N, 60°E) B (30°N, 80°E) C (0°, 60°E)
   D (60°S, 40°E) E (30°N, 20°W)
2. (a) 20° (b) 60° (c) 100° (d) 20° (e) 40°
3. (a) 80° (b) 50° (c) 140° (d) 90°
4. Malindi
5. When they are position θ° north or south of the Equator.
6. (0°, 37°), Lake Turkana, 4°
7. (a) AOG, CGO, CRO, BOH,GOR

Exercise 6.2

1.	(a) 5 005 km	(b) 2 068 nm
2.	(a) 13 694 km	(b) 7 388 nm
3.	(a) 36 719 km	(b) 19 808 nm
4.	(a) 40 034 km	(b) 21 597 nm

5. 5 728 nm
6. 48.2°
7. (a) 6 840 km (b) 3 690 nm
8. (a) 2 233 km (b) 1 205 nm
9. 69.8°E
10. (a) 6 434 km (b) 3 471 nm
11. P = 60°S
12. 47.14 cm
13. 2 800 km (2 s.f.) or 15 115.2 nm
14. 3 300 nm or 6 100 km (2 s.f.)
15. 74°S (2 s.f)
16. 81°N or S (2.s.f)
17. 24°E (2 s.f)
18. 8 898 km (nearest km)

Exercise 6.3

1.	(a)	(i) 50 knots	(ii) 100 knots	
		(iii) $1405\frac{5}{7}$ knots	(iv) 2 knots	
	(b)	(i) 120 nm	(ii) 1 800 nm	(iii) 324.8 nm
2.	(a)	1528 hours	(b) 1336 hours	(c) 0804 hours
	(d)	1328 hours	(e) 1952 hours	(f) 1020 hours
	(g)	1244 hours	(h) 0600 hours	(i) $1121\frac{1}{3}$ hours
3.	(a)	1 457 km/h	(b) 775.5 knots	
4.	(a)	825.4 knots	(b) 1 530 km/hr	
5.		37.5 hours		
6.	(a)	200 knots	(b) 370 km/h (2 s.f.)	
7.	(a)	513 knots for B (nearest knot)	(b) 5.00 a.m.	
		690 knots for A (nearest knot)	0500 hrs	
8.		Y is 48.6°N		
9.	(a)	2 000 km	(b) 101.5°W	
10.		Time for A is 11.78 hours		

11. $\alpha = 33.3°$ S

Time for B is 17.52 hours; longitude 2.4° W

Mixed Exercise 2

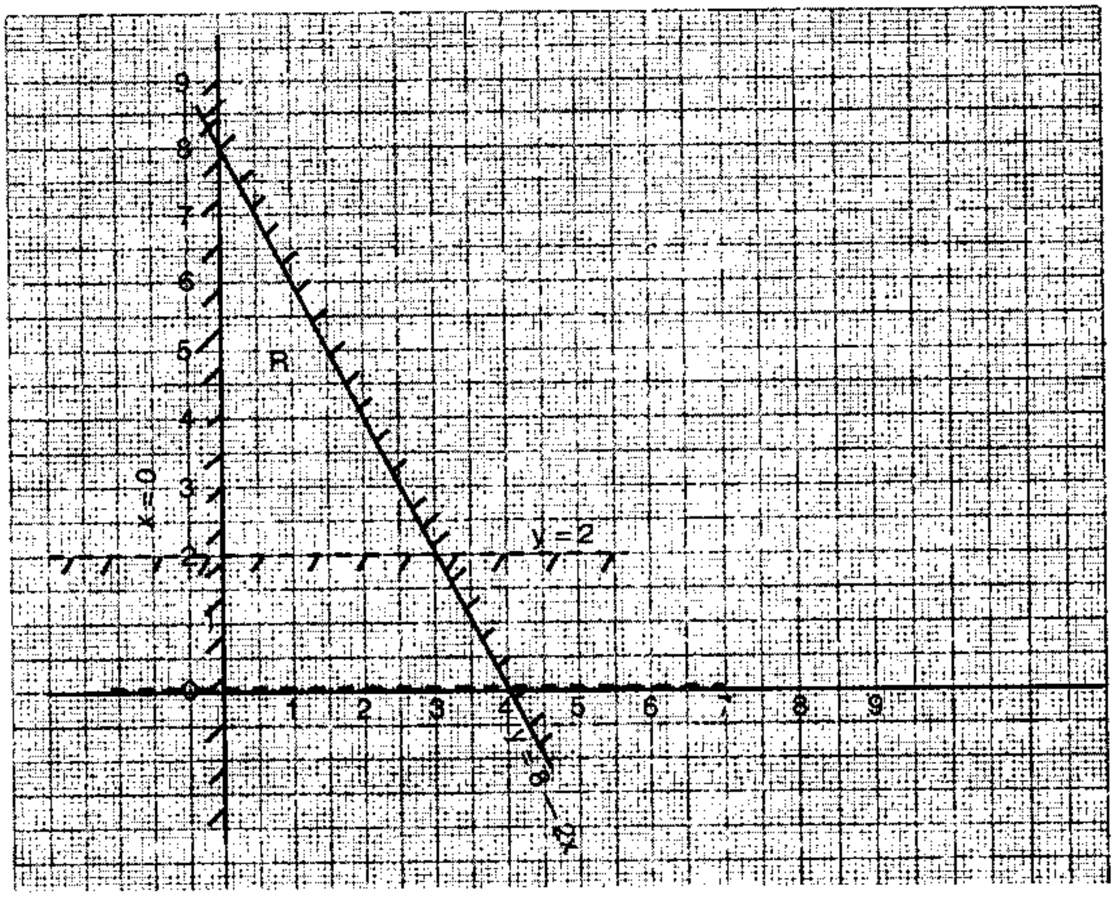
1. (a) $x^{12} + 12x^{11} + 66x^{10} + 220x^9 + 495x^8 + \ldots$

   (b) $1 - 13x + 78x^2 - 286x^3 + 715x^4 \ldots$

   (c) $256 + 1\,024x + 1\,792x^2 + 1\,792x^3 + 1\,120x^4 + \ldots$

   (d) $512 - 1\,152x + 1\,152x^2 - 672x^3 + 256x^4 + \ldots$

2. 0° or 360°

3. 

4. Given $\sin 2\theta = 2\sin\theta\cos\theta$, to prove that $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$,

   $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

   But $\sin^2\theta + \cos^2\theta = 1$, and $2\sin\theta\cos\theta = \sin 2\theta$

   $\therefore (\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$.

5. (a) DF = 14.14 (b) 45°

6. 3 889 km

7.

	Amplitude	*Period*
3 sin (2x + 30°)	3	π or 180°
sin 2x	1	π or 180°

(c) 3 sin (2x + 30°) leads sin2x by 30°

8. – 108 864

9. $1 + \frac{1}{2}x + \frac{5}{48}x^2 + \frac{5}{432}x^3 + ...$; 3.75

10. Check for correct table of values and graph; θ = 20°

11. $\sqrt{3}$

12. 2 160 nm, latitude 176°W

13. Translation $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$, stretch of 3 units parallel to y-axis.

$\frac{1}{2}$ unit parallel x-axis.

14. 30°, 150°

15. 69.4°, 249.4°

16. (a) $1\,024x^5 - 1\,280x^4y + 640x^3y^2 - 160x^2y^3 + 20xy^4 - y^5$

(b) $1 + \frac{3}{2}x + \frac{15}{16}x^2 + \frac{5}{16}x^3 + \frac{15}{256}x^4 + \frac{3}{512}x^5 + \frac{1}{4096}x^6$

17. – 4 320

18. 1 294.272864 (6 d.p.)

19. 0°, 180°, 360°

20. (a) Amplitude $\frac{1}{2}$ (b) Period 2π or 360°

21. – 97.2°, 180° or 262.8°

22. (a) $\sin x = \frac{3}{5}, \cos x = \frac{4}{5}$

(b) $\sin x = \frac{11}{\sqrt{265}}, \cos x = \frac{12}{\sqrt{265}}$

(c) $\sin x = \frac{15}{17}, \cos = \frac{-8}{17}$

23. (a) 0. 984 (3 d.p.) (b) 988.719 (3 d.p.) (c) 1.062 (3 d.p.)

24. $\frac{1}{6561x^8} - \frac{8}{729x^6} + \frac{28}{81x^4} - \frac{56}{9x^2} + 70 - 504x^2 + 2268x^4 - 5832x^6$
$+ 6561x^8$

(b) $30\sqrt{3}$

(c) $-9\,072\sqrt{6}$

25. (a) 60° (b) 54°

26. 1814.4 km

27. (a) $\tan A = \frac{1}{3}$ (b) $\frac{9\sqrt{3}+7\sqrt{5}}{4}$

28 (a) $\tan(A + B + C) = \tan\{(A + B) + C\}$

$$= \frac{\tan(A+B)+\tan C}{1 - \tan(A+B)\tan C}$$

$$\text{But } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{So, } \tan\{(A+B) + C\} = \frac{\frac{\tan A + \tan B}{1 - \tan A \tan B} + \tan C}{1 - \frac{\tan A + \tan B}{1 - \tan A \tan B}\tan C}$$

$$= \frac{\frac{\tan A + \tan B + \tan C(1 - \tan A \tan B)}{1 - \tan A \tan B}}{\frac{1 - \tan A \tan B - \tan C(\tan A + \tan B)}{1 - \tan A \tan B}}$$

$$= \frac{\tan A + \tan B + \tan C(1 - \tan A \tan B}{1 - \tan A \tan B - \tan C(\tan A + \tan B)}$$

$$= \frac{\left(\frac{2}{5} + \frac{1}{2}\right) + \frac{1}{4}\left(1 - \frac{2}{5} \times \frac{1}{2}\right)}{1 - \frac{2}{5} \times \frac{1}{2} - \frac{1}{4}\left(\frac{2}{5} + \frac{1}{2}\right)}$$

$$= \frac{\frac{4}{5} + \frac{1}{4} \times \frac{9}{10}}{\frac{4}{5} - \frac{1}{4} \times \frac{9}{10}}$$

$$= \frac{11}{10} \div \frac{23}{40}$$

$$= \frac{44}{23}$$

(b) 36.7°

29. $10\sqrt{2}$; 25.1°
30. 0° or 360°
31. (a) $\frac{1}{1296}$ (b) $\frac{24}{1296}$ (c) $\frac{60}{1296}$
32. 17.5°
33. 10.52 cm
34. $1 - 22x + 178x^2 - 575x^3$
35. (a) 3 114 km (b) 1 680 nm
36. 290.7 cm
37. 525 : 18
38. 1022.4 km/h
39. 25.4°

Chapter Seven

LINEAR PROGRAMMING

The concept of linear programming is new to the learner. However, the learner has met linear inequalities, a pre-requisite concept for this topic, in Book Two.

Objectives

By the end of the topic, the learner should be able to:

(i) form linear inequalities based on real life situations.
(ii) represent the linear inequalities on a graph.
(iii) solve and interpret one optimum solution of the linear inequalities.
(iv) apply linear programming to real life situations.

Time: Twenty one lessons.

Teaching/ Learning Activities

Forming Linear Inequalities

- The teacher should introduce the formation of linear inequalities, as in the students' book.
- The learner should be taken through examples 1 and 2.
- The learner to do exercise 7.1.

Solutions of Linear Inequalities

- The learner should be guided through the analytical solution of linear inequalities, as in examples 3 and 4.
- The teacher should discuss graphical solutions of linear inequalities, as in examples 5 and 6.
- The learner to do exercise 7.2.

Optimisation

- The teacher should discuss optimum solution of linear ineqaulities, as illustrated in the students' book.
- The learner to do exercise 7.3.

Evaluation

- The teacher should give a written test on linear programming.

Answers

Exercise 7.1

1. $b > 4\,000$
   $3b \leq 16\,000$

2. $x > y$
   $x + y > 100$
   $y \geq 20$

3. $x \geq 119$
   $y < 76.5$
   $y > 0$

4. $x \geq 25$
   $x > 10$

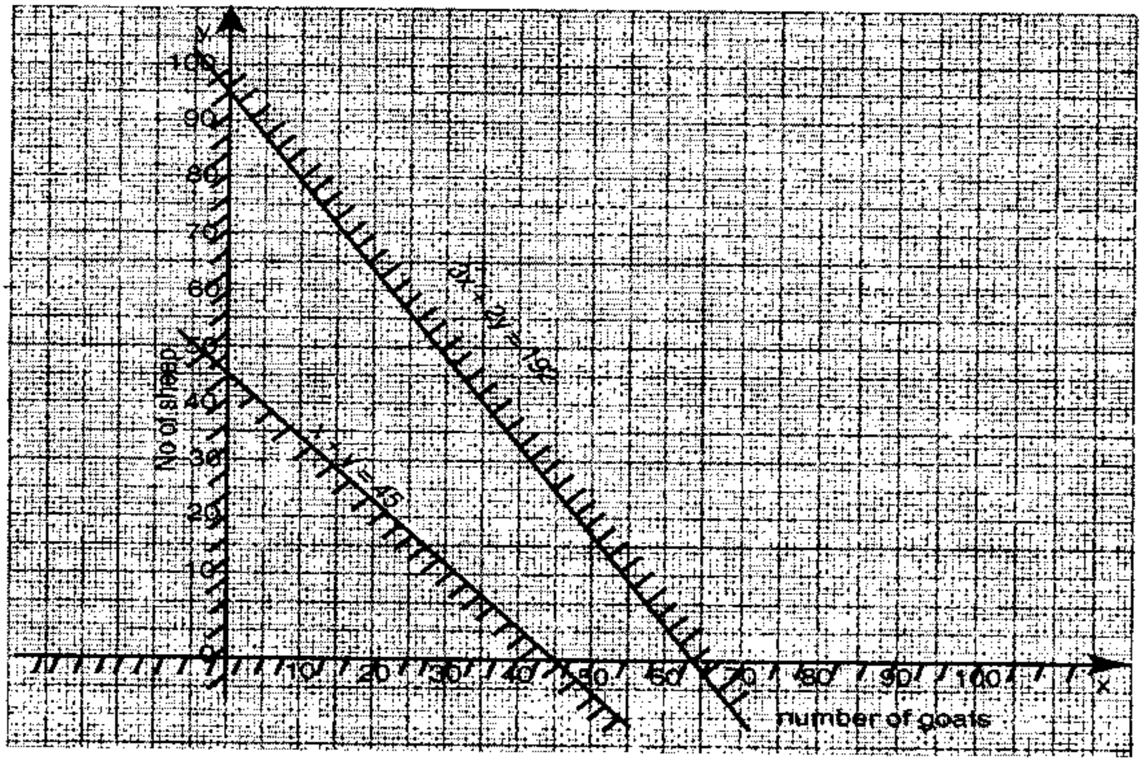
5. $2a > 3b$
   $ab \geq 17$
   $a > b$

6. $x + y \leq 7$
   $5x + 12y \leq 96$
   $x > 0$
   $y > 0$

7. $x + y \leq 400$
   $5x + 3y \geq 1\,500$
   $x > 0, y > 0$

Exercise 7.2

1. Maximum salary that can be paid to the less paid employee is sh. 13 000.
2. $3\frac{1}{3} < x < 6$
3. $x > 15$
   $x \leq 30$
   Range $15 < x \leq 30$
4. Let the number of groups of exercise books be x and that of class readers be y.
   Possible purchases:
   (4, 0) (5, 0) (6, 0) (7, 0) (8, 0) (9, 0) (10, 0)
   (4, 1) (5, 1) (6, 1) (7, 1) (8, 1) (9, 1)
   (4, 2) (5, 2) (6, 2) (7, 2) (8, 2)
5. There are very many possibilities (see figure below)
   $x > 0, \quad y > 0, \quad x + y > 45, \quad 3x + 2y \leq 192$



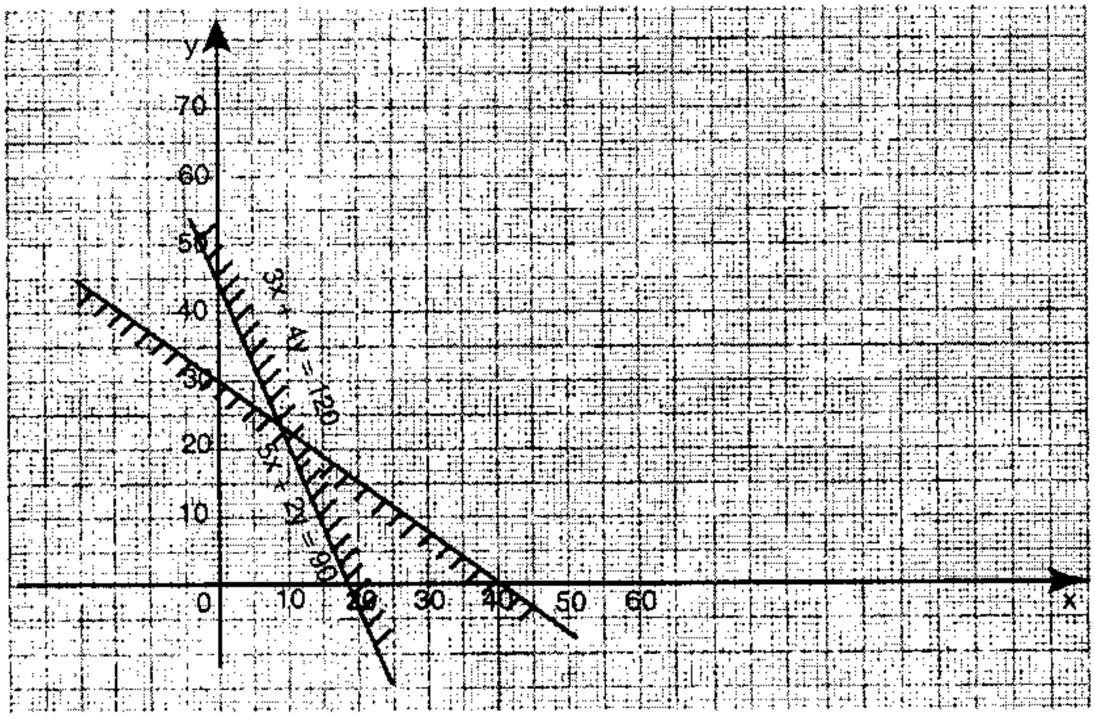
6.

A	1	2	3
B	3	2	2

7.

Jane	*Clara*
11	12
11	13
11	14
12	13
12	14
12	15
13	14
13	15
13	16
14	15

8. Let the number of suits to be made be x and that of dresses be y.
$5x + 2y < 90$, $3x + 4y > 0$, $y > 0$



(1, 40)	(1, 41)	(1, 42)	(2, 29)	(2, 30)	(2, 31)	(2, 32)	(2, 33)	(2, 34)	
(2, 35)	(2, 36)	(2, 37)	(2, 38)	(2, 39)	(3, 30)	(3, 31)	(3, 32)	(3, 33)	(3, 34)
(3, 35)	(3, 36)	(3, 37)	(4, 28)	(4, 29)	(4, 30)	(4, 31)	(4, 32)	(4, 33)	(4, 34)
(5, 27)	(5, 28)	(5, 29)	(5, 30)	(5, 31)	(5, 32)	(6, 26)	(6, 27)	(2, 28)	(6, 29)
(7, 25)	(7, 26)	(7, 27)	(8, 24)						

Possible amounts $2\,400x + 900y = k$

9.

Trunk, local	Trunk, local	Trunk, local	Trunk, local	Trunk, local	Trunk, local
(x, y)	(x, y)	(x, y)	(x, y)	(x, y)	(x, y)
(0, 12)	(1, 12)	(2, 12)	(3, 12)	(4, 12)	(5, 12)
(0, 13)	(1, 13)	(2, 13)	(3, 13)	(4, 13)	(5, 13)
(0, 14)	(1, 14)	(2, 14)	(3, 14)	(4, 14)	(5, 14)
(0, 15)	(1, 15)	(2, 15)	(3, 15)	(4, 15)	(5, 15)
(0, 16)	(1, 16)	(2, 16)	(3, 16)	(4, 16)	(5, 16)
(0, 17)	(1, 17)	(2, 17)	(3, 17)	(4, 17)	(5, 17)
(0, 18)	(1, 18)	(2, 18)	(3, 18)	(4, 18)	(5, 18)
(0, 19)	(1, 19)	(2, 19)	(3, 19)	(4, 19)	(5, 19)
(0, 20)	(1, 20)	(2, 20)	(3, 20)	(4, 20)	(5, 20)
(0, 21)	(1, 21)	(2, 21)	(3, 21)	(4, 21)	(5, 21)

(0, 22)	(1, 22)	(2, 22)	(3, 22)	(4, 22)	(5, 22)
(0, 23)	(1, 23)	(2, 23)	(3, 23)	(4, 23)	(5, 23)
(0, 24)	(1, 24)	(2, 24)	(3, 24)	(4, 24)	(5, 24)
(0, 25)	(1, 25)	(2, 25)	(3, 25)	(4, 25)	(6, 13)
(0, 26)	(1, 26)	(2, 26)	(3, 26)	(6, 14)	(6, 15)
(0, 27)	(1, 27)	(2, 27)	(6, 16)	(6, 17)	(6, 18)
(0, 28)	(1, 28)	(6, 19)	(6, 20)	(6, 21)	(6, 22)
(0, 29)	(6, 23)	(7, 15)	(7, 16)	(7, 17)	(7, 18)
(7, 19)	(7, 20)	(7, 21)	(7, 22)	(8, 18)	(8, 17)
(8, 19)	(8, 20)	(8, 21)	(9, 19)	(9, 20)	

Possible expenditure $12x + 8y = k$

10. Let number of excersice books be x and the number of pencils y.

(x, y)	(x, y)	(x, y)	(x, y)	(x, y)
(1, 11)	(1, 19)	(2, 18)	(3, 17)	(4, 18)
(1, 12)	(2, 11)	(2,19)	(3, 18)	(4, 19)
(1, 13)	(2, 12)	(3, 11)	(4, 12)	(5, 15)
(1, 14)	(2, 13)	(3, 12)	(4, 13)	(5, 16)
(1, 15)	(2, 14)	(3, 13)	(4, 14)	(5, 17)
(1, 16)	(2, 15)	(3, 14)	(4, 15)	(5, 18)
(1, 17)	(2, 16)	(3, 15)	(4, 16)	(6, 18)
(1, 18)	(2, 17)	(3, 16)	(4, 17)	(6, 19)

Possible expenditure; $5x + 20y = k$

11. (30, 10) (30, 20) (30, 25) (30, 30) (30, 40) (30, 45) (40, 10) (40, 20) (40, 30) (40, 35) (35, 35)

Cost function $= (4 \times 50) + 45\ [\ (x - 1) + (y - 1)]$

$$= 200 + 45\ (x + y - 2)$$

A few of the possible costs are:

1 910, 2 360, 2 585, 2 810, 3 260, 3 485

12. (9, 5) (10, 4) (10, 5) (11, 4) (11, 5) (12, 3) (12, 4) (12, 5)
(13, 3) (13, 4) (13, 5) (14, 2) (14, 3) (14, 4) (14, 5) (15, 1)
(15, 2) (15, 3) (16, 1) (16, 2) (17, 1)

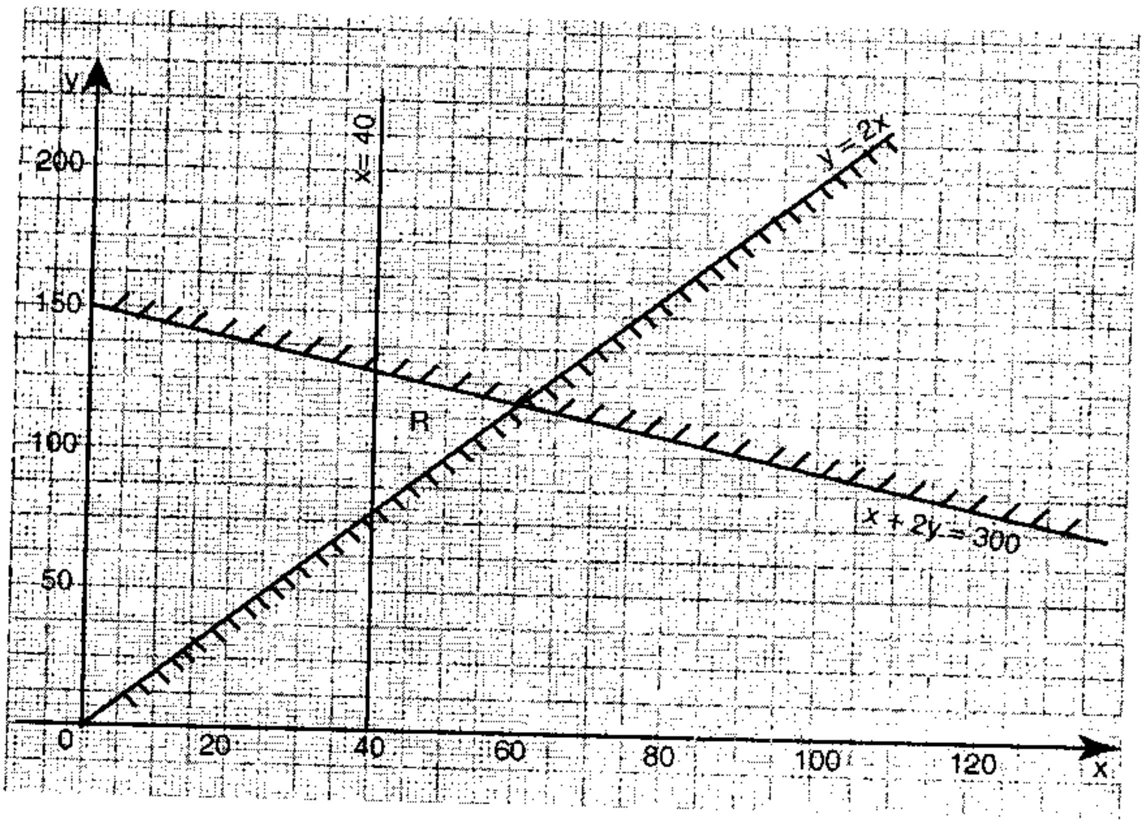
Corresponding costs:

sh. 320 sh. 340 sh. 350 sh. 370 sh. 380 sh. 390 sh. 400
sh. 410 sh. 420 sh. 430 sh. 440 sh. 440 sh. 450 sh. 460
sh. 470 sh. 460 sh. 470 sh. 480 sh. 490 sh. 500 sh. 520

13. There are many possible combinations, see graph on page 54. Let number of sh. 5 coins be x and sh. 10 coins be y.
Some of the combinations are listed below.

(40, 80) (40, 89) (40, 98) (40, 107) (40, 116) (41, 82) (41, 92) (41, 102)
(40, 81) (40, 90) (40, 99) (40, 108) (40, 117) (41, 83) (41, 93) (41, 103)
(40, 82) (40, 91) (40, 100) (40, 109) (40, 118) (41, 84) (41, 94) (41, 104)
(40, 83) (40, 92) (40, 101) (40, 110) (40, 119) (41, 85) (41, 95) (41, 105)
(40, 84) (40, 93) (40, 102) (40, 111) (40, 120) (41, 86) (41, 96) (41, 106)
(40, 85) (40, 94) (40, 103) (40, 112) (40, 121) (41, 87) (41, 97) (41, 107)
(40, 86) (40, 95) (40, 104) (40, 113) (40, 122) (41, 88) (41, 98) (41, 108)
(40, 87) (40, 96) (40, 105) (40, 114) (40, 123) (41, 89) (41, 99) (41, 109)
(40, 88) (40, 97) (40, 106) (40, 115) (40, 124) (41, 90) (41, 100) (41, 110)
(40, 125) (41, 91) (41, 101) (41, 111)

(41,112) (41,123) (42,88)(42,99) (42,110)(42,121)(43, 88) (43,99) (43,110)
(41,113) (41,124) (42,89)(42,100)(42,111)(42,122)(43,89) (43,100) (43,111)
(41,114) (41,125) (42,90)(42,101)(42,112)(42,123)(43,90) (43,101) (43,112)
(42,115) (41,126) (42,91)(42,102)(42,113) (42,124)(43,91) (43,102) (43,113)
(42,116) (41,127) (42,92)(42,103) (42,114)(42,125) (43,92) (43,103) (43,114)
(41,117) (41,128) (42,93) (42,104)(42,115) (42,126) (43,93) (43,104) (43,115)
(41,118) (41,129) (42,94)(42,105) (42,116) (42,127) (43,94)(43,105) (43,116)
(41,119) (42,84) (42,95)(42,106) (42,117) (42,128) (43,95) (43,106) (43,117)
(41,120) (42,85) (42,96)(42,107) (42,118) (42,129) (43,96) (43,107) (43,118)
(41,121) (42,86) (42,97)(42,108) (42,119) (43,86) (43, 97) (43,108) (43,119)
(41,122) (42,87) (42,98)(42,109) (42,120) (43,87) (43,98) (43,109) (43,120)



Exercise 7.3

1. 20 2. 2 3. 100 4. 92

5. Let number of cameras be x and the number of briefcases be y.
   $x + y \geq 120$, $x \geq 30$, $y \geq 60$
   Objective function: $40x + 25y = k$
   Point for minimum value of objective function (30, 90)
   Hence 30 cameras and 90 briefcases should be displayed to minimise the cost of display.

6. Let type A cakes be x and type B cakes y.
   Constraints:
   Number of eggs $3x + 6y \leq 40$
   Amount of sugar $8x + 3y \leq 40$
   $x > 0$, $y > 0$
   Maximise $25x + 20y$
   Type A cakes 3
   Type B cakes 5

7. Let the width be x m and the length y m.
   Constraints:

$x \geq 5$, $2y < 3x$, $2x + y \leq 100$
Maximise $x + y$
Maximum value of length + width = 72 [point (29, 43)]

8. Let the number of bottles of juice be x and the cakes y.
Constraints: $15x + 10y \leq 300$ $(3x + 2y \leq 60)$
$y \geq 15$, $x > 0$
Bottles of juice are 10

9. Let the number of round stools be x and the number of rectangular schools be y.
Constraints:
$2x + y \leq 80$
$x \geq 15$, $y \geq 10$
$6x + 5y \leq 300$
Maximise $80x + 60y$
Round stools 25
Rectangular stools 30

10. Let brand 1 be x kg and brand 2 be y kg
Constraints:
$2x + y \geq 9$, $x + y \geq 7$, $x + 2y \geq 10$, $x + 3y \geq 12$
$x > 0$, $y > 0$
Minimise $10x + 14y$

Brand 1	4 kg
Brand 2	3 kg

Minimum cost for the mixture is sh. 82.

Chapter Eight

DIFFERENTIATION

This is a new topic to the learner. However, in Book Three, the learner was exposed to average and instantaneous rates of change, which are introductory activities to differentiation.

Objectives

By the end of the topic, the learner should be able to:

(i) find average rates change and instantaneous rates of change.

(ii) find the gradient of a curve at a point using tangent.

(iii) relate delta notation to rates of change.

(iv) find the gradient function of a function of the form $y = x^n$ (where n is a positive integer).

(v) define:
- derivative of a function.
- derived function of a polynomial.

(vi) determine the derivative of a polynomial.

(vii) find the equations of tangents and normals to a curve.

(viii) sketch a curve.

(ix) apply differentiation in calculating velocity and acceleration.

(x) apply differeniation in finding maxima and minima of functions.

Time: Nineteen lessons.

Teaching/ Learning Activities

Average and Instantaneous Rates of Change

- The teacher should involve the learner in the revision of gradient of a curve at a point.
- The learner should be involved in determining the average rate of change between two points on a curve, as in the students' book.
- The teacher should guide the learner to find the gradient of a curve at a point, as in the students' book.

Gradient of $y = x^n$

- The learner should be introduced to the gradient of $y = x^n$, as in the students' book.
- The teacher should guide the learner to make generalisation on finding the gradient function of $y = x^n$, as in table 8.2 of the students' book.
- The learner should be led to relate the delta notation to the rates of change, as in the students' book.
- The teacher should guide the learner through example 1.

The Derivative of a Polynomial

- The teacher should guide the learner to find the derivative of a polynomial, as in the students' book.
- The learner should be guided to make a generalisation on finding the derivative of a polynomial, as in the students' book.
- The teacher should lead the learner through examples 2, 3 and 4.
- The learner to do exercise 8.1.

Equations of Tangents and Normals to a curve

- The teacher should discuss the gradient of a curve at a point and the gradient of a tangent to the curve at that point.
- The teacher should take the learner through example 5.
- The teacher should define the normal to a curve at a point, as in the students' book.
- The learner should be led through example 6.
- The learner to do exercise 8.2.

Stationary Points

- The teacher should discuss stationary points, as in the students' book, identifying the points of minima, maxima and inflection.
- The teacher should lead the learner through examples 7 and 8.
- The learner do exercise 8.3 to introduce curve sketching.
- The teacher should guide the learner through example 9.
- The learner to do exercise 8.4.

Application of Differentiation in Calculation of Velocity and Acceleration

- The teacher should discuss the application of differentiation in calculating velocity, as in the students' book.
- The learner should be led through example 10.
- The teacher should guide the learner on the application of

differentiation in calculating acceleration, as in the students' book.

- The teacher should guide the learner through example 11.
- The learner to do exercise 8.5.

Maxima and Minima

- The teacher should guide the learner through examples 12 and 13.
- The learner to do exercise 8.6.

Additional Hints

- The teacher can use the second derivative to test on maxima and minima.
- Maximum displacement is attained when v = 0.
- Maximum velocity is attained when v = 0.

Evaluation

- The teacher should test the concept of differentiation by giving enough practice.

Answers

Exercise 8.1

1. (a) $y' = 6x$ (b) $y' = 2$ (c) $y' = 6x^2 - 3$
2. (a) $y' = 5x^4$ (b) $8x^7$ (c) 4 (d) 9
   (e) 7 (f) $\frac{-1}{2}$ (g) 0 (h) 0
   (i) 0 (j) −5
3. (a) $12x^3$ (b) $4x^3 - 6x$ (c) $20x^3 - 10x^4 - 21x^2$
   (d) $-10x^{19} - x^8$ (e) $28x^6 + \frac{6}{7}x^5 - 3x^2$ (f) 3
   (g) $4x^3 + 3$ (h) $\frac{1}{2} - 3x^2$
4. (a) $6x - 7$ (b) $14x - \frac{1}{2}$ (c) $4x^3 - 12x^2 + x$
   (d) 3 (e) 0 (f) 0
5. (a) $\frac{1}{2}x^4 + 21x^2 - 4x$ (b) $3x^2 - 2x - 1$
   (c) $16t^3 - \frac{1}{3}t + 7$ (d) $45t^4 + 3t^2 - 3$
   (e) $60r^2 + r^3$ (f) $r^{23} - \frac{1}{4}r^9$
6. (a) 1 (b) 0 (c) 4 (d) −7

(e) -8

7. (a) $(3,\ 20)$ (b) $\left(1,\ 1\frac{2}{3}\right)$ and $\left(-2,\ 1\frac{1}{3}\right)$

(c) $(0, 0)$, $\left(1,\ \frac{3}{4}\right)$ $\left(5, -31\frac{1}{4}\right)$

8. (a) 1 (b) $3x^2 - 6x - 1$ (c) $1 - \frac{3}{x^2} - \frac{4}{x^3} + \frac{15}{x^4}$

(d) $\frac{1}{5} - \frac{5}{x^2}$ (e) $16x + 10$ (f) $2x - 1$

(g) $-\frac{1}{x^2} - 1$ (h) $\frac{7}{6}x^{\frac{1}{6}} - 1$ (i) $\frac{-2}{x^3} - \frac{3}{x^4} - \frac{1}{x^2}$

Exercise 8.2

1. (a) $y = 5x - 2$ (b) $y = 24x - 31$

(c) $y = -3x + 1$ (d) $y = 0$

2. (a) $y = \frac{-1}{5}x + \frac{16}{5}$ (b) $y = \frac{-1}{24}x + 17\frac{1}{12}$

(c) $y = \frac{1}{3}x + 1$ (d) $y = 0$

3. $y = 3x - 2$ 4. $y = 3\frac{1}{2} - \frac{1}{4}x$

5. (a) $y = 3$ (b) $x = 1$

6. $y = {}^{-}1$ at $(0, 1)$ and $(2, 1)$, $x = 0$ at $(0, {}^{-}1)$ and $x = 2$ at $(2, {}^{-}1)$

7. $(8,\ 36)$; $y = \frac{-1}{8}x + 37$

8. $(4, 11)$ Equation of tangent $y = 5x - 9$

Equation of normal $y =$ $y = \frac{-1}{5} + \frac{59}{5}$

Exercise 8.3

1. (a) $(0,\ 5)$ Point of inflection.

(b) $(1,\ 1)$ Minimum point.

(c) $\left(\frac{-7}{4}, \frac{169}{8}\right)$ Maximum point.

(d) $(0, 2)$ Maximum, $(1, 3)$ Minimum, $({}^{-}1, 3)$ minimum

(e) $\left(\frac{3}{2},\ \frac{59}{16}\right)$ Maximum point.

2. (a) $\left(\frac{-5}{2}, \frac{315}{4}\right)$ Maximum point.

$(1, -7)$ Minimum point.

(b) $(0, 36)$ Point of inflection, $(0.24, 35.7)$ Point of inflection.

$(6.24, -508.4)$ Minimum point.

(c) $(4, -68)$ Minimum point.

$\left(\frac{1}{2}, 17\frac{3}{4}\right)$ Maximum point.

3. $y = \frac{1}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 + 7$

$y' = x^4 + 2x^3 + x^2$

$= x^2 (x^2 + 2x + 1)$

At turning point, $y' = 0$

$x^2 (x + 2x + 1) = 0$

$x^2 (x + 1)^2 = 0$

$x = 0$, or $x = -1$

x	$\frac{-1}{2}$	0	1
y'	+ve	0	+ve

When $x = 0$, the curve has a point of inflection.

x	-2	-1	$\frac{-1}{2}$
y'	+ve	0	+ve

When $x = -1$, the curve has a point of inflection.

Exercise 8.4

1. (a) – (h) Check for the correct sketch.

Exercise 8.5

1. (a) 40 m (b) 20 ms^{-2} (c) $t = 1$ s and $t = 3$ s
2. (a) 3 ms^{-2}, 1 ms^{-2} (b) -4 ms^{-2}, 32 ms^{-2}
   (c) 9 ms^{-2}, 19 ms^{-2} (d) 0 ms^{-2}, -10 ms^{-2}
   (e) 24 ms^{-2}, 38 ms^{-2}
3. (a) (i) -27 ms^{-2} (ii) -69 ms^{-2} (b) $\frac{-1}{14}$ s
4. (a) $t = 0$ s, $t = 2$ s (b) $t = 1$ s

5. Displacement, 4 m
Velocity 0 ms^{-1}
Acceleration 6 ms^{-2}

(b) Displacement, 8 m.
Velocity 10 ms^{-2}
Acceleration 18 ms^{-2}

(c) Displacement 308 m
Velocity 280 ms^{-1}
Acceleration 198 ms^{-2}

6. (a) -143 ms^{-2} (b) $\frac{2}{5}$ s (c) -64 ms^{-2}

7. (a) Height 43 m
Velocity 24 ms^{-1}
Acceleration -36 ms^{-2}

(b) Height 49 m
Velocity $-12ms^{-1}$
Acceleration -36 ms^{-2}

(c) Height 19 m
Velocity $-$ 48 ms^{-2}
Acceleration -36 ms^{-2}

(d) Height 51 m
Velocity 0 ms^{-1}
Acceleration -36 ms^{-2}

Exercise 8.6

1. 6.003 m
2. $A = x(9 - x)$ m^2; length 4.5 m, width 4.5 m.
3. Radius 5 cm, height 5 cm
4. $S = \frac{1}{3}t^3 - \frac{3}{2}t^2$

$$\frac{dS}{dt} = v = t^2 - 3t$$

$$\frac{dv}{dt} = \text{accel} = 2t - 3$$

For velocity, minimum $\frac{dv}{dt} = 0$

$2t - 3 = 0$

$$t = \frac{3}{2}$$

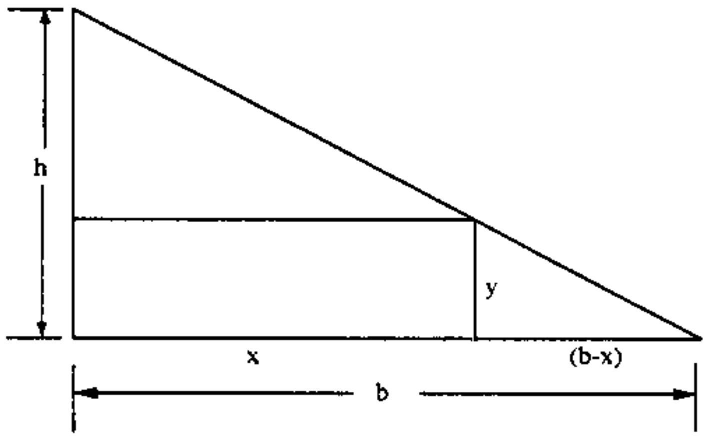
t	1	$\frac{3}{2}$	2
$\frac{dv}{dt}$	-ve	0	+ve
	\	—	/

Velocity is minimum at $t = \frac{3}{2}$.

Velocity = 0 ms^{-1}

5. Maximum area is 1 250 m^2

6.



From Similarity;

$$\frac{h}{y} = \frac{b}{(b-x)}$$

$$y = h - \frac{hx}{b}$$

Area of rectangle, A = xy.

$$= x\left(h - \frac{hx}{b}\right)$$

$$= xh - \frac{hx^2}{b}$$

$$\frac{dA}{dx} = h - \frac{2xh}{b}$$

For maximum area, $\frac{dA}{dx} = 0$

$$h - \frac{2hx}{b} = 0$$

$$x = \frac{b}{2}$$

Area of rectangle, $A = \frac{b}{2}\left(h - \frac{hx}{b}\right)$

$$= \frac{bh}{2} - \frac{hxb}{4}$$

$$=\frac{2bh-bh}{4}$$

$$=\frac{bh}{4}$$

$$=\frac{1}{2}\times\frac{bh}{2}$$

But $\frac{bh}{2}$ is the area of triangle

Therefore, area of rectangle = $\frac{1}{2}$area of the triangle

7. $V = x^3 + 2x^2 - 3x$

$$\frac{dV}{dx}=3x^3 + 4x - 3$$

For minimum volume, $\frac{dV}{dx}=0$

$3x^2 + 4x - 3 = 0$

$$x = \frac{-4 \pm \sqrt{16+36}}{6}$$

$$=\frac{-4 \pm \sqrt{52}}{6}$$

$$=\frac{-4 \pm \sqrt{4x13}}{6}$$

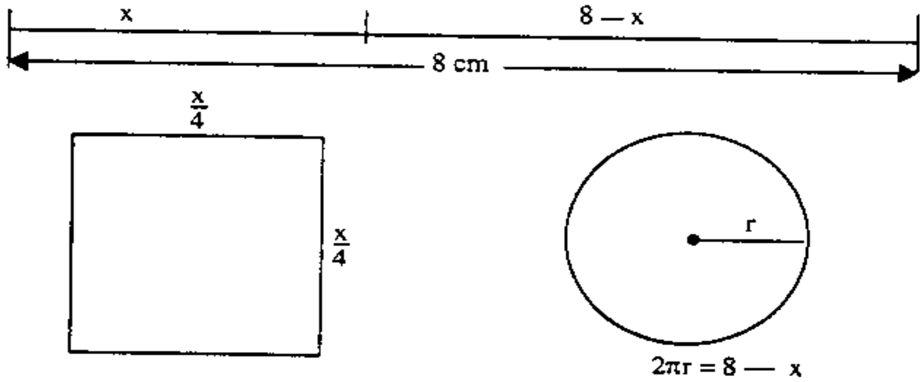
$$=\frac{-4 \pm 2\sqrt{13}}{6}$$

$$=\frac{-2 \pm \sqrt{13}}{3}$$

$$x = \frac{-2-\sqrt{13}}{3} \text{ or } \frac{-2+\sqrt{13}}{3}$$

Ignore $\frac{-2-\sqrt{13}}{3}$, since there is no negative dimension.

So, $x = \dfrac{-2 + \sqrt{13}}{3}$ cm

8. 

Area of the square, $A_1 = \frac{x^2}{16}$ cm^2

Area of circle, $A_2 = \pi r^2$

$$= \pi \left(\frac{8-x}{2\pi}\right)^2$$

$$= \frac{(8-x)^2}{4\pi}$$

Total area, $A = A_1 + A_2$

$$= \frac{x^2}{16} + \frac{(8-x)^2}{4\pi}$$

$$= \frac{\pi x^2 + 4(64 - 16x + x^2)}{16\pi}$$

$$= \frac{\pi x^2 + 256 - 64x + 4x^2}{16\pi}$$

$$\frac{dA}{dx} = \frac{1}{16\pi}[2x\pi - 64 + 8x]$$

For minimum area, $\frac{dA}{dx} = 0$

$$\frac{1}{16\pi}(2\pi x + 8x - 64) = 0$$

$$2\pi x + 8x - 64 = 0$$

$$2\pi x + 8x = 64$$

$$x = \frac{64}{2\pi + 8}$$

$$= \frac{32}{\pi + 4}$$

But $r = \dfrac{8 - x}{2\pi}$

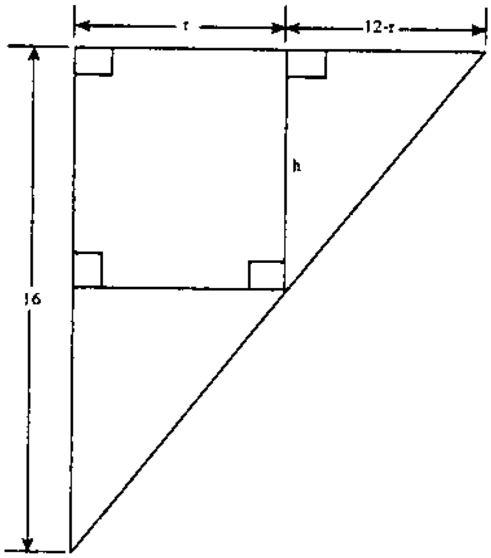
$$r = \frac{8 - \dfrac{32}{(\pi + 4)}}{2\pi}$$

$$= \frac{8\pi + 32 - 32}{\pi + 4} \div 2\pi$$

$$= \frac{8\pi}{2\pi(\pi + 4)}$$

$$= \frac{4}{\pi + 4}$$

9.



From Similarity;

$$\frac{16}{h} = \frac{12}{(12-r)}$$

$$h = \frac{16(12-r)}{12}$$

$$= \frac{4(12-r)}{3}$$

Volume of cylinder, $V = \pi r^2 h$

$$= \pi r^2 \frac{4}{3}(12-r)$$

$$= \frac{4}{3}\pi r^2(12-r)$$

$$= 16\pi r^2 - \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 32\pi r - 4\pi r^2$$

For minimum volume, $\frac{dV}{dr} = 0.$

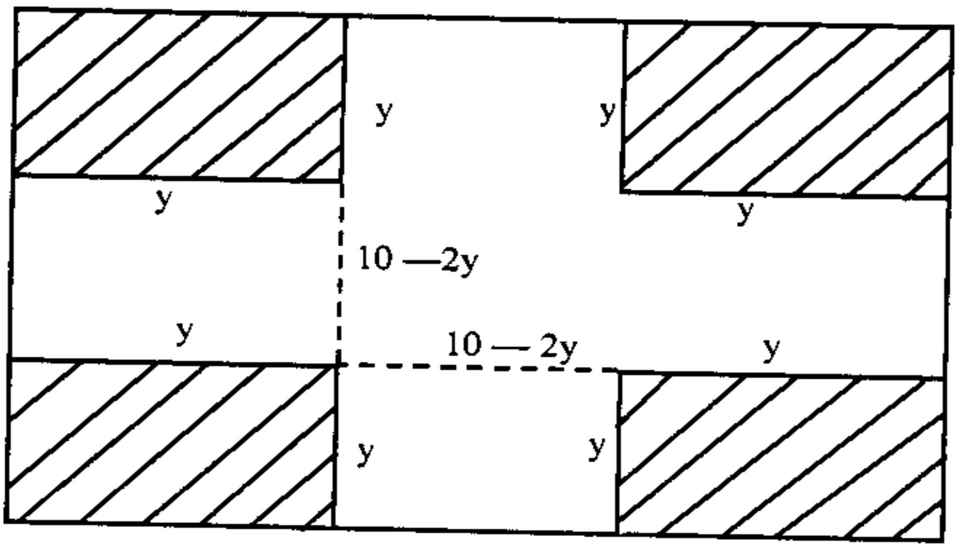
$32\pi r - 4\pi r^2 = 0$

$4\pi r(8 - r) = 0$

$r = 0$ or $r = 8$

Ignoring $r = 0$, radius $r = 8$ cm.

10.



Area of square metal $= 100\ m^2$. Each side is 10 m.

Volume of cuboid, V $= (10 - 2y)(10 - 2y)y$

$= y(100 - 40y + 4y^2)$

$= 100y - 40y^2 + 4y^3$

$$\frac{dV}{dy} = 12y^2 - 80y + 100$$

For volume maximum, $\frac{dV}{dy} = 0$.

$12y^2 - 80y + 100 = 0$

$4(3y^2 - 20y + 25) = 0$

$3y^2 - 20y + 25 = 0$

$$y = \frac{+20 \pm \sqrt{400 - 300}}{6}$$

$$= \frac{20 \pm \sqrt{100}}{6}$$

$$= \frac{20 \pm 10}{6}$$

$y = 5$ or $y = \frac{10}{6} = \frac{5}{3}$

$y = 5$ is unrealistic.

For maximum volume, height of tank is $\frac{5}{3}$ m.

Chapter Nine

AREA APPROXIMATION

In Book One, the learner dealt with the estimation of area by counting squares. In this topic, the learner will be exposed to more accurate methods of approximating area.

Objectives

By the end of the topic, the learner should be able to:

(i) approximate the area of irregular shapes by the counting technique.
(ii) derive the trapezium rule.
(iii) apply trapezium rule to approximate areas of irregular shapes.
(iv) apply trapezium rule to estimate area under a curve.
(v) derive the mid-ordinate rule.
(vi) apply mid-ordinate rule to approximate area under a curve.

Time: Ten lessons.

Teaching/ Learning Activities

Using Counting Technique to Approximate Area

- The teacher should discuss the approximation of areas by counting technique, as in the students' book.
- The learner to do exercise 9.1.

Approximating Area by Trapezium Method

- The teacher should guide the learner in deriving the trapezium rule, as in the students' book.
- The learner should be led through examples 1 and 2.
- The learner to do exercise 9.2.

The Mid-ordinate Rule

- The learner should be led to derive the mid-ordinate rule, as in the students' book.
- The teacher should guide the learner through examples 3 and 4.
- The learner to do exercise 9.3.

Project

The teacher should involve the learner in approximating areas of such irregular shapes as footprints palm-prints, patches on the floor or wall by tracing the same and transferring to squared paper. Accuracy should be emphasised by dividing the areas into as many regular shapes as possible.

Evaluation

- Give a written test on trapezium rule and mid-ordinate rule.

Answers

Exercise 9.1

1. (a) 583 200 km^2 (b) 75 600 km^2
   (c) 165 600 km^2 (d) 5 472 km^2

Exercise 9.2

1. (a) 2.75 sq. units	(b) 13.5 sq. units	(c) 4.18 sq. units
2. (a) 38.5 cm^2	(b) 36.72 cm^2	(c) 4.62 %
3. 22	4. 55.2 ha	5. 114.7 m
6. 34.6 km	7. 13.94 J	8. (a) 26 (b) 26.5
9. 2 425 m^2(10 trapezia)		(10) 14.4 m^3

11.

x (rads)	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
2x (rads)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
sin 2x	0	0.7071	1.000	0.7071	0

Area = 0.95 sq. units.

Exercise 9.3

1. 106 sq. units	2. 27 sq. units	3. 48 sq. units
4. 10.75 sq. units	5. 20.12 sq. units	6. 23.9 ha
7. 510 m	8. 8.7 m^3 (6 strips)	9. 317.3 m^3
10. 800 m^2	11. 3.61×10^{-2} As (Coulombs)	

Chapter Ten

INTEGRATION

This topic is new to the learner. However, the learner has dealt with differentiation and area approximation, which are related to integration.

Objectives

By the end of the topic, the learner should be able to:

(i) interpret integration as a reverse process of differentiation.

(ii) relate integration notation to sum of areas of trapezia under a curve.

(iii) integrate a polynomial.

(iv) apply integration in finding the area under a curve.

(v) apply integration in kinematics.

Time: Nineteen lessons.

Teaching/ Learning Activities

Reverse Differentiation

- The learner should be introduced to integration as the reverse of differentation, as in the students' book.
- The learner should be guided to establish the rule of integration using example 1.
- The learner should be guided through example 2.
- The learner to do exercise 10.1.
- The teacher should guide the learner through examples 3 and 4.
- The learner to do exercise10.1 (continued).

Definite and Indefinite Integrals

- The teacher should discuss the relation of integration notation to the sum of areas of trapezia.
- The teacher should guide the learner to appreciate definite and indefinite integrals as in the students' book.
- The learner should be guided through example 5.
- The learner to do exercise 10.2

Area Under a Curve

- The teacher should discuss how to find the exact area under a curve, as in the students' book.
- The learner should be led through examples 6, 7, 8 and 9.
- The learner to do exercise 10.3.

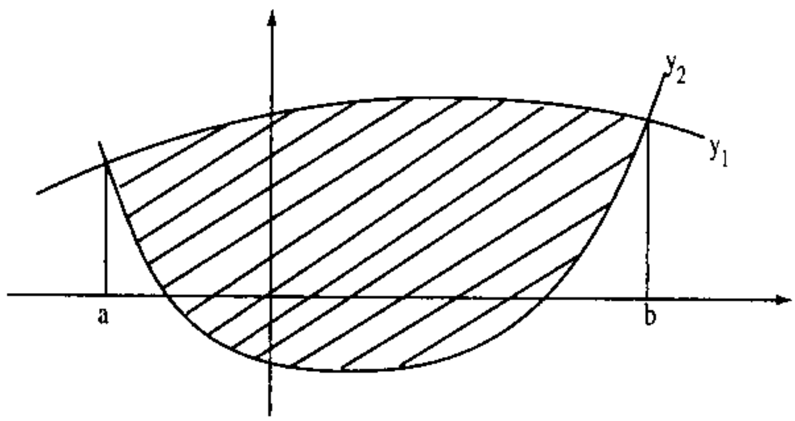
Application in Kinematics

- The teacher should discuss application of integration in kinematics, as in the students' book.
- The learner should be guided through examples 10, 11 and 12.
- The learner to do exercise 10.4.

Additional Hints

- If a region is bounded by two curves y_1 and y_2 as shown below, then the area of the enclosed region is evaluated from;

$$\int_a^b (y_1 - y_2)\, dx$$



- Sketching of curves to get the boundaries of the required region should also be emphasised.

Evaluation

- Oral and written tests on integration should be given.

Answers

Exercise 10.1

1. (a) $\frac{3}{2}x^2 + c$ (b) $7x + c$

   (c) $x^2 + 4x + c$ (d) $\frac{x^3}{3} + x^2 + c$

(e) $\frac{x^6}{6} + c$

(f) $\frac{x^8}{8} + c$

(g) $\frac{-x^{-4}}{4} + c$

(h) $\frac{-3}{5}x^5 + c$

(i) $\frac{5}{6}x^{-6} + c$

(j) $\frac{5}{4}x^4 - \frac{4x^3}{3} + c$

(k) $\frac{3}{5}x^5 + x^{-1} + 3x + c$

(l) $\frac{9}{4}x^4 + \frac{2x^{-3}}{3} + x^3 + c$

(m) $2x^4 + x^3 + 2x^2 + 3x + c$

(n) $-x^{-1} + c$

(p) $-x - \frac{x^3}{3} + c$

(q) $\frac{1}{3x^3} - \frac{1}{2x^2} + c$

(r) $\frac{8}{3}x^{\frac{3}{2}} + \frac{1}{10}x^5 + x^3 + c$

2. (a) $y = ax^4 + c$

(b) $y = x + \frac{4}{x} - \frac{2}{x^2} + c$

(c) $y = x - \frac{16}{3}x^3 + c$

(d) $y = \frac{x^3}{3} + 3x^2 + 9x + c$

(e) $y = -\frac{1}{x} - c$

3. (a) $y = x^3 - \frac{x^2}{4} + 4x + c$

(b) $y = x^4 + x^3 + x^2 - 2x + c$

(c) $y = x - \frac{1}{x} - \frac{1}{2x^2} + c$

(d) $y = 3x - \frac{3}{x} - \frac{1}{x^2} - \frac{1}{3x^3} + c$

(e) $y = \frac{3}{2x^2} - \frac{4}{3x^3} - \frac{2}{x} - x + c$

4. (a) $y = \frac{x^3}{3} + x^2 + c$

(b) $y = \frac{3}{4}x^4 - \frac{5}{6}x^3 + 7x + c$

(c) $y = 5x - \frac{1}{x} + c$

(d) $y = \frac{10}{3}x^{\frac{3}{2}} + x^3 + 2x + c$

(e) $y = \frac{9}{4}x^{\frac{4}{3}} + 4x^{\frac{5}{4}} - \frac{7}{4}x^4 + 9x + c$

(f) $y = \frac{16}{5}x^{\frac{5}{4}} + \frac{1}{2}x + c$

(g) $y = \frac{65}{6}x^{\frac{6}{5}} - \frac{1}{12x^4} + 8x + c$

5. $A = r^4 + r^3 + 4r + 13$

6. $y = x^3 + 2x - 4$

7. $y = \frac{x^3}{3} - \frac{x^2}{2} + \frac{x}{4} + \frac{1}{8}$

8. (a) $r = \frac{t^4}{2} - t^2 + \frac{t^3}{3} - t + 1$ (b) 38.5

Exercise 10.2

1. (a) 114 (b) 80 (c) 95 (d) 432

2. (a) $x^3 - 2x^2 + 2x + c$ (b) $4x - \frac{x^4}{4} + c$

   (c) $\frac{x^5}{5} + x^4 - 3x^3 + \frac{x^2}{2} + c$ (d) $\frac{t^3}{3} + t^2 + t + c$

   (e) $\frac{t^4}{4} - \frac{2}{3}t^3 + \frac{3}{2}t^2 - 4t + c$

3. (a) 150 (b) $603\frac{1}{3}$ (c) $631\frac{2}{3}$ (e) -2

   (f) $\frac{-4}{3}$ (g) $-10\frac{2}{3}$

Exercise 10.3

1. (a) $\frac{1}{6}$ sq. units (b) $1\frac{1}{3}$ sq. units

   (c) $84\frac{1}{4}$ sq. units (d) 7.15 sq.units

   (e) $12\frac{2}{3}$ sq.units (f) 171.5 sq. units

2. 9.48 sq. units

3. $1\frac{1}{3}$ sq. units

4. $2\frac{1}{3}$ sq. units

5. (a) $11\frac{1}{4}$ sq. units (b) $\frac{4}{3}$ sq. units

   (c) $21\frac{1}{3}$ sq. units (d) $3\frac{1}{12}$ sq. units

   (e) 0.5 sq. units

6. (a) $85\frac{1}{3}$ sq. units

7. (a) $\frac{1}{6}$ sq. units (b) $\frac{5}{6}$ sq. units

   (c) 0.75 sq. units

Exercise 10.4

1. (a) $666\frac{2}{3}$ m (b) $29\frac{1}{3}$ m (c) 34 m

   (d) $86\frac{2}{3}$ m (e) 1.5 m (f) $594\frac{2}{3}$ m

2. (a) $100\ ms^{-1}$ (b) $34\frac{2}{3}\ ms^{-1}$ (c) $0\ ms^{-1}$

   (d) $106\frac{2}{3}\ ms^{-1}$ (e) $\frac{19}{60}\ ms^{-1}$ (f) $924\frac{3}{4}\ ms^{-1}$

3. $S = 3t^3 + 3t$; 114m

4. (a) $21.5\ ms^{-1}$ (b) 29.93

5. $a = -6t$

   $v = 3t^3 + c$

   $c = 20$

   $v = 20 - 3t^2$

   $0 = 20 - 3t^2$

   $t^2 = \frac{20}{3}$

   $t = \sqrt{\frac{20}{3}}$

6. (a) 3 seconds

   (b) $S = \frac{t^3}{3} - \frac{t^4}{12} + c$

   $S = \theta$ at $t = \theta \Rightarrow c = \theta$

   $\therefore S = \frac{t^3}{3} - \frac{t^4}{12}$

   After 3 seconds;

   $S = \frac{1}{3}(3)^3 - \frac{(3)^4}{12} = \frac{9}{4}$ m

7. (a) $\frac{dv}{dt} = a$

   $v = at + c$

   When $+ t = 0$, $c = u$

   $\therefore v = u + at$

   (b) $\frac{dS}{dt} = u + at$

$S = ut + \frac{1}{2}at^2 + c$

Since S = 0 when t = 0, c = 0

$\therefore\ S = ut + \frac{1}{2}at^2$

8. (a) 2 seconds
   (b) 20 ms^{-1}
9. $v = 10t + 24$
   $S = 5t^2 + 24t$
10. (a) $v = -10t + 16$ (b) $S = \frac{-5}{2}t^2 + 16t + 0.5$
    (c) 19.7 m

Mixed Exercise 3

1. (a) $6\frac{2}{3}$ (b) $34\frac{2}{3}$ (c) $-88\frac{1}{3}$
2. a = 1, b = 4, c = 0
3. (a) $\frac{1}{3}$ (b) $y = \frac{x^2}{2} - x + 1$
4. Velocity = 479 ms^{-1}
   Acceleration = 488 ms^{-2}
5. (a) $40x^7 + \frac{5}{4}x^4 - 4x^3$
   (b) $-2x^5 - 2x^{11}$
6. 4
7. (a) $-3\frac{3}{4}$ (b) $\frac{1}{2}$ (c) -21
8. (a) $a = \frac{1}{2}$ (b) $S = \frac{t^3}{3} + \frac{3t^2}{2}$
   $S = \frac{t^2}{4} - t$
   (c) $a = 30t + 6$ (d) $S = \frac{t^4}{4} - t^3 + \frac{3t^2}{2} - t$
   $a = 3t^2 - 6t + 3$
9. (a) $y' = 4x + 3$ (b) $y' = 3x^2 + 8x - 3$
   (c) $y' = 4x^3 + 15x^2 - 4x + 1$
10. $\frac{11}{12}$
11. (a) $a = 6t - 10$ (b) $a = 24t - 6$

(c) $a = 3t - 6$ (d) $a = 5t^3 - 150t^4 + 6t + 2$

12. $y = x^3 - \frac{x^2}{2} + x - \frac{13}{2}$

13. (a) $\left(2, \frac{-22}{3}\right)$, minimum; $\left(-3, \frac{27}{2}\right)$, maximum

(b) $\left(2, \frac{14}{3}\right)$, maximum; $\left(3, 4\frac{1}{4}\right)$, minimum

(c) $\left(2\frac{1}{2}, \frac{-1}{4}\right)$, minimum

(d) $\left(2\frac{1}{2}, \frac{1}{4}\right)$, minimum

(e) $(0,0)$, minimum; $\left(-4, 10\frac{2}{3}\right)$; maximum

14. 70, $(x = 15, y = 40)$

15. (a) $\frac{x^3}{3} - \frac{3}{2}x^{-2} + 2x^2 - 4x + c$

(b) $\frac{-3}{2x^2} + \frac{4x^5}{5} + \frac{5}{4x^4} + \frac{6}{7}x^7 + c$

(c) $-\frac{2}{x} + \frac{1}{x^3} - \frac{1}{x^4} + \frac{1}{x^5} + c$

16. (a) $y' = 6x^2 - 2x + 3$

(b) $y' = 6x^2 - 2x + 2$

(c) $y' = 10x - 2$

(d) $y' = 4x + 6$

17. $t = 1$ sec; $v = 0\ ms^{-1}$; $a = 1\ ms^{-1}$

18. (a) $6\frac{1}{3}$ (b) 12

(c) $256\frac{1}{4}$ (d) $\frac{4b^3}{3} + \frac{5b^2}{2} - \frac{4a^3}{3} - \frac{5a^2}{2}$

19. (a) $x - x^2 - \frac{8}{7}x^{\frac{7}{2}} + c$ (b) $\frac{x^3}{9} - \frac{x^2}{4} + 4x + c$

(c) $\frac{2x^3}{9} - \frac{x^2}{10} + \frac{1}{4}x + c$ (d) $\frac{x^8}{8} + c$

20. $v = 4t + 5$, $S = 2t^2 + 5t$

21. $85\frac{1}{3}$

22. (a) 42 (b) 41.89

23. $y = 11x - 16$

24. 15.5; 15; % error = $3\frac{1}{3}\%$

25. 9 (3, 3) 9 (3, 3)

26. $\frac{dy}{dx} = 4x + 3$; (2, 15) $\frac{dy}{dx} = 6x - 1$; (2, 9)

27. $155\frac{3}{4}$

28. 10 she-goats, 8 he-goats.

29. $3\frac{1}{6}$

30. $y = 37x - 46$, $37y + x = 1\,038$

31. 1.867 sq. units

32. 30 sq. units

33. $\frac{1}{2}\text{km}^2$

34. 6.4259 sq. units

35. (a) $2t^3 - 7t^2 + 7t - 2$

(b) (i) $t = \frac{1}{2}, t = 1, t = 2$

(ii) When $t = 0$, $s = -2\text{m}$, $v = 7\text{ ms}^{-1}$, $a = -14\text{ ms}^{-1}$

$t = 2$, $S = 4\text{ m}$, $v = 3\text{ ms}^{-1}$, $a = 10\text{ ms}^{-2}$.

(iii) $t = \frac{7 \pm \sqrt{7}}{6}$ seconds

36. (i) 19 sq. units

(ii) 18 sq. units

Error = 5.6 %

37. (a) $t = 0$ and $t = \frac{2}{5}$ seconds

When $t = 0$, $a = -2\text{ ms}^{-2}$

When $t = \frac{2}{5}$, $a = 2\text{ ms}^{-2}$

(b) $26\frac{2}{3}\text{m}$

38. 6.065 sq. units

39. 1.395

40. (a) $41\frac{1}{3}$ (b) 2 ms^{-2} (c) 7.045 seconds

Revision Exercise 1

1. $\angle QPR = 104°29'$, $\angle PRQ = 46°\ 34'$, $\angle PQR = 28°\ 57'$

2. A translation of $\begin{pmatrix} -8 \\ -8 \end{pmatrix}$; inverse $\begin{pmatrix} 8 \\ 8 \end{pmatrix}$.

3. (a) Period 360° Amplitude 2 (b) Period 180° Amplitude 1 (c) Period 360° Amplitude $\frac{1}{3}$

   (a) Period 120° Amplitude $\frac{1}{3}$ (b) Period 720° Amplitude 1 (c) Period 1 080° Amplitude 1

4. (a) $x = 6$, $y = 0$ (b) $x = 2$, $y = 0$ (c) $x = 2$, $y = 6$ (d) $x = 0$, $y = 2$

5. (a) $\frac{1}{2}$ (b) $\frac{1}{26}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$

6. 14.63 cm

7. (a) $x > 3\frac{1}{3}$ (b) $x < 1\frac{1}{4}$ (c) $x < 3\frac{1}{4}$

8. (a) $\frac{9}{8}$ (b) $\frac{27}{73}$

9. (a) A′ (1, 2), B′ (2, 2), C′ (2, 3), D′ (0, 4)

   (b) A′ (2, 3), B′ (1, 3), C′ (1, 2), D′ (3, 1)

   (c) A′ (1, 8), B′ (1, 7), C′ (2, 7), D′ (3, 9)

10. $\lambda = 1 + \sqrt{6}$ and $\lambda = 1 - \sqrt{6}$

11. AC = 25 cm, BC = 14 cm

12. A = 170, n = −1.1

13. 1 796 nm (4 s.f.)

14. Two major arcs with a common chord AB.
    Angle subtended at the centre by chord AB = 80°.
15. $n = 2.3 \pm 0.01, A = 2$

Revision Exercise 2

1. sh. 720
2. (a) 5 : 1 (b) 10 : 3 (c) 10 : 9
3. $x = -1$
4. $P'(4, 4)$, $Q'(12, 4)$, $R'(8, 12)$
   An enlargement, centre (0, 0), scale factor 2.
5. (a) 54.46 (b) 0.00002025 (c) 5.434
   (d) 68.58 (e) 0.00005242 (f) 0.000005837
6. (a) 40.5 ms^{-1}
   (b) $\frac{1}{12}$ s
7. $x = 0$ or $x = 1$
8. (a) $\frac{1}{2}$ (b) 360°
9. 68 m
10. (a) $\frac{5}{16}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$
11. (a) $x < 0$ (b) $4 < x < 6$ (c) $12 < x < 14$
(d) $x < \frac{-3}{5}$
12. (a) $y + x = 4$
    (b) A quarter turn about (2, 2)
13. (a) $-35y^3x^4$ (b) $108\,864y^5x^3$ (c) $\frac{224}{27}y^5x^4$
    (d) $\frac{-1792}{9}x^3$ (e) $\frac{3}{5}x^2$
14. 6.2 units
15. 2.05 m

Revision Exercise 3

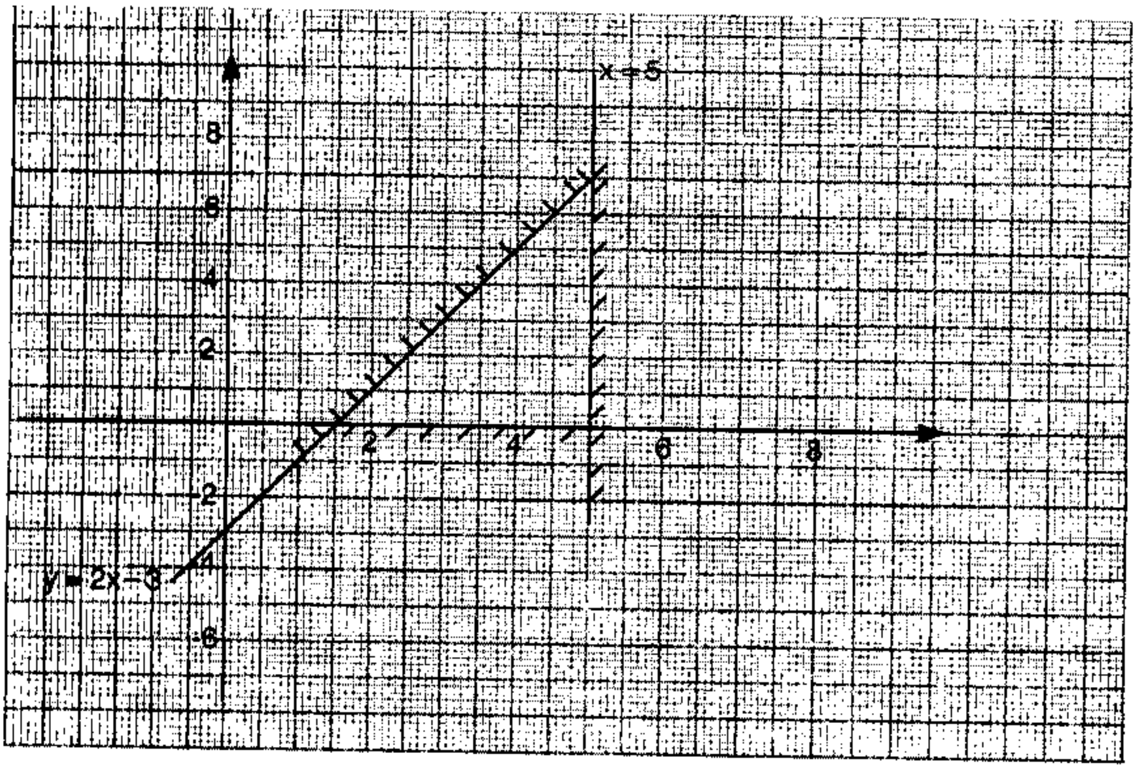
1. Check for correct construction.
   $LM = 12.5 \pm 0.1$cm
   $\angle LMN = 103° \pm 0.1°$

2. 0.04555

3. (a) $\frac{2}{3}$ (b) $\frac{1}{3}$

4. $\mathbf{a}\begin{pmatrix}-1\\2\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}0\\-1\end{pmatrix}$

5.



6. 156
7. 30°
8. 2.11
9. (a) Check for the correct ogive.
   (i) 37.5 ± 0.5, 20 ± 0.4(1 s.s.)

   (b) $\frac{162}{540}$

10. $20\frac{5}{6}$ sq. units
11. 19.8 cm, 17.6°
12. (a) Check for correct curve.
    (b) (i) $x = -1$ or $x = 1.8$ or $x = -0.8$
    (ii) $x = 1.8$
    (iii) $x = -1.8$ or $x = 0.8$ or $x = 1$

(iv) x = 0 or x = 1.6 or x = –1.6

13.(a)

$x°$	0	30	60	90	120	150	180	210	240	270	300	330	360
$\cos x°$	1	0.87	0.5	0	–0.5	–087	–1	–0.87	–0.5	0	0.5	0.87	1
$2\cos x$	2	1.75	1.0	0	–1	–1.75	–2	–1.75	–1	0	1	1.75	2
$2x$	0	60	120	180	240	300	360	420	480	540	600	660	720
$\sin 2x$	0	0.87	0.87	0	–0.87	–0.87	0	0.87	0.87	0	–0.87	–0.87	0
$y = 2\cos x + \sin 2x$	2	2.61	1.87	0	–1.87	–2.61	–2	–0.87	–0.13	0	–0.13	–0.87	2

14. Turning points $\left(\frac{-1}{3}, \frac{-4}{3}\right)$ ± 1 small square; minimum turning point.

(a) x = 1 ± 1 small square or x = 0.3 ± 1 small square.

(b) x = – 0.8 ± 1 small square or x = 0.4 ± 1 small square.

(c) x = – 4 ± 1 small square or 3.4 ± 1 small square.

15. $x = \frac{^-13}{18}$, $y = \frac{1}{6}$, $z = \frac{17}{36}$

Revision Exercise 4

1. a = 13, b = 8, c = -2
2. 93°
3. (a) (6, 3) and (4, –1)

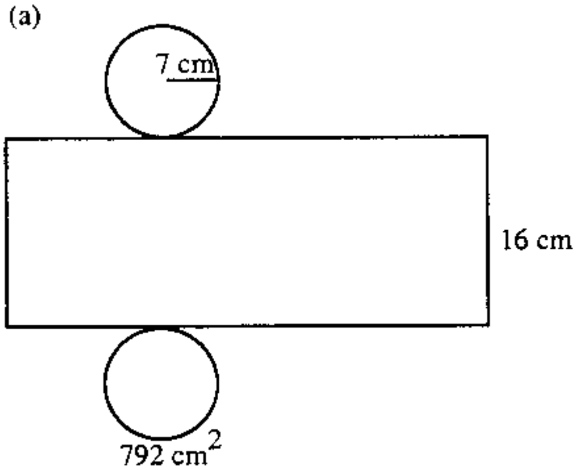
   (b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
4. $\mathbf{AM} = \frac{2}{3}\mathbf{a} + \mathbf{c}$, $\mathbf{BN} = \frac{1}{3}(\mathbf{a} - 2\mathbf{c})$; 3 : 4
5. $\log_{10}\left(\frac{x^3 y^{\frac{1}{3}}}{z^2}\right)$
6. (a) 25.96 cm^2 (b) 9.238 cm^2
7. $\frac{15}{29}$
8. $25\frac{1}{3}$ sq. units

9. $r = 6.30$ cm, $h = 12.6$ cm
10. (a) $a^8 + 8ab + 28a^6b^2 + 56\,a^5b^3 + \ldots$
    (b) $a^7 - 7a^6b + 21a^5b^2 - 35\,a^4b^3 + \ldots$
    (c) $a^6 + 12a^5b + 60a^4b^2 + 160a^3\,b^3 + \ldots$
11. 13.79 cm
12. $41°25'$
13. (a) 12.81 cm (b) $27°56'$
14. (a) $\cos A = -\cos(180 - B)$
    (b) $QS^2 = 20 - 16\cos P$; $QS^2 = 34 - 30\cos R$
    (i) $107°\,42'$ (ii) 4.989 cm
15. (b) (i) 3.4 (ii) 3.9 (iii) 15.6
    (iv) -42.9 (v) -3.6

Revision Exercise 5

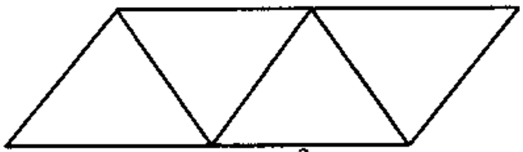
1. $\frac{25}{3}$
2. (a) $P''(1, -3)$, $Q''(-1, -3)$, $R''(-1, -2)$ and $S''(1, -2)$
   (b) A rotation of 180° about (1, 0) or an enlargement of scale factor -1 centre (1, 0)
3. $\frac{4}{17}$
4. (a), (b), (c) - Check for accuracy of graphs.
5. (a) 16 m (b) $8\ ms^{-1}$ (c) $-8\ ms^{-2}$
6. 20, 22 and 24
7. 5
8. $y + x = -1$
9. 18
10. 7.421 cm
11. Ksh. 70 926
12. $x = 9.014$ cm, $y = 40$ cm, z- = 20 cm
13. 45°
14. (a) 210° (b) 60° or 210°
15.

(a)



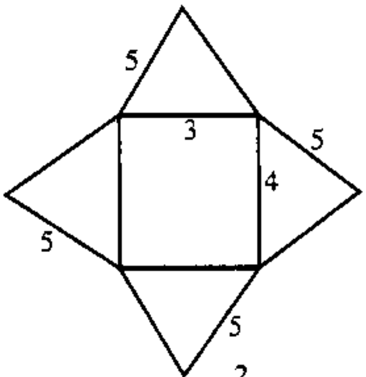
792 cm^2

(b)



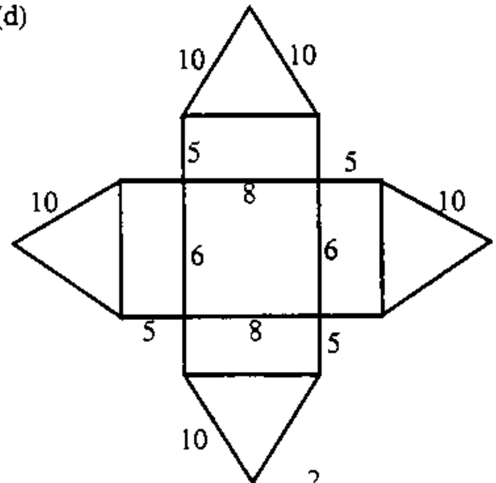
62.35 cm^2

(c)



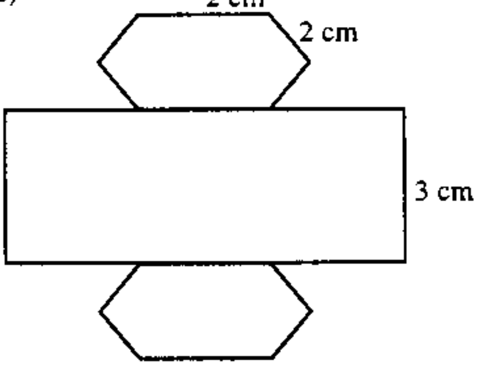
44.64 cm^2

(d)



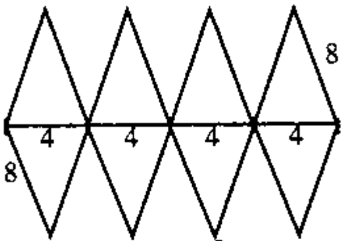
318.6 cm^2

(e)



48 cm^2

(f)



123.9 cm^2

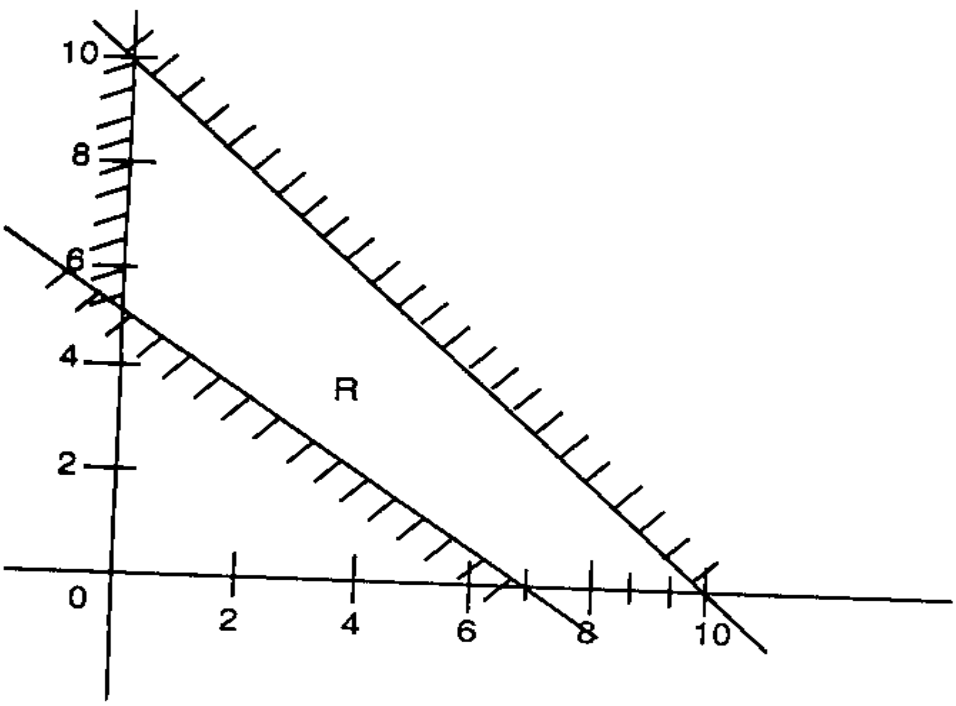
Revision Exercise 6

1. (a) 0.064, 0.0638, 0.06376

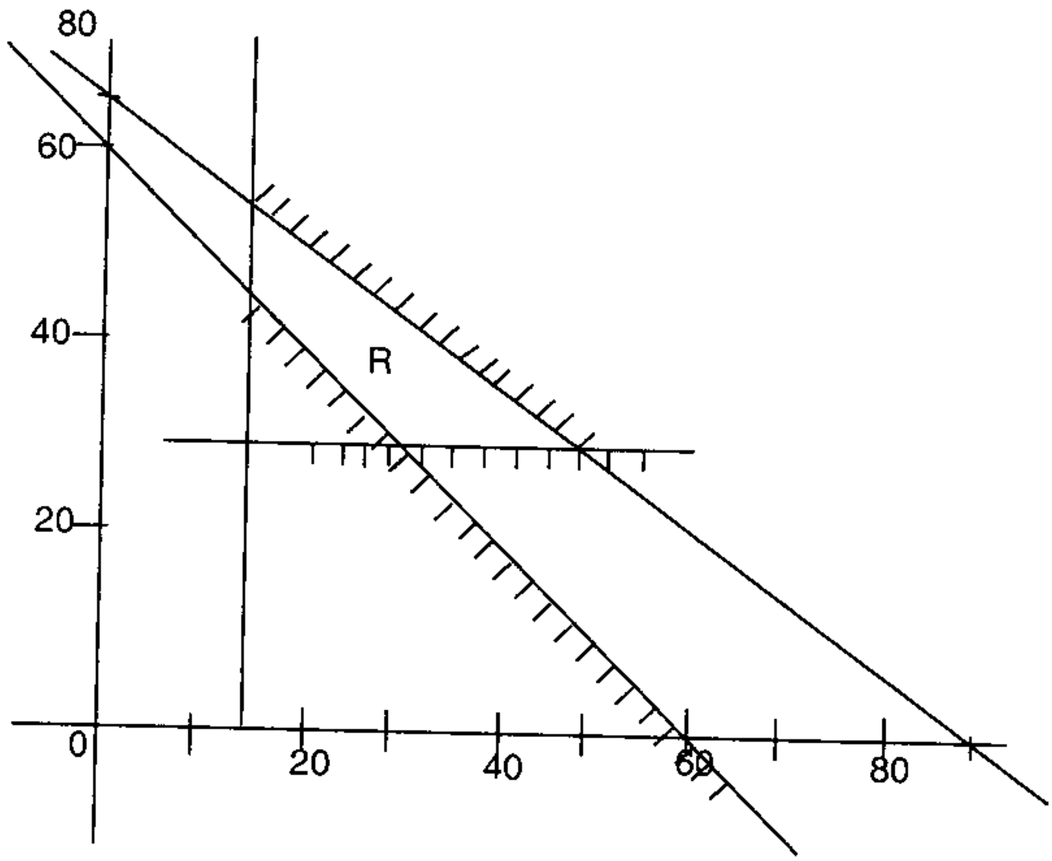
3.0, 3.00, 3.002

0.26, 0.257, 0.2569
0.00, 0.000, 0.0001

2. 31.82 cm
3. $y = 7x + 9, \; 7y + x = 37$
4. $x = 5, y = 4$
5. $\begin{pmatrix} \frac{5}{2} \\ \frac{-5}{2} \end{pmatrix}$
6. (a) 93.53 cm^2 (b) 0.0928
7. (a) 13 (b) 322 (c) 2 977
8. (a) 64°52′ (b) 25°08′ (c) 55°20′′
9. $y + x = 5, \; y = x + 1$
10. (a) 5.882 % (b) 12 : 1
11. $\frac{2x + y}{x + 2y}$
12. 11 by 7
13. (a)



(b)



14. (a) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{-2}{3} & \frac{5}{3} \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$

(d) $\begin{pmatrix} \frac{-4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$

15. sh. 1 000 000

Revision Exercise 7

1. R (6, 4)
2. (a) Angle YBZ = 50°
   Angle YDZ = 130°
   Angle CZB = 65°

3. $\frac{1}{9}$

4. ± 4

5. 8.5625 units^2

6. 8.9 %

7. 4.1 cm

8. $83\frac{1}{3}$ units^2

9. sh. 600

10. 60.03

11. (a) 30 (b) 120

12. Angle SPQ = 60°
Angle SQP = 40°
Angle RTO = 75°

13. (a) 3.97 cm^2
(b) 9.863 cm

14. (a) VP = 11.02 cm
(b) 65.20°
(c) 76.83°

15.

t	0.5	2	2.5	3	3.5	4.5
h	17.5	41.5	41.5	37.5	29.5	1.5

(a) (i) 14.62 (ii) 33.18 (iii) 25.18

(b) 0.6 sec and 3.9 sec

(c) 42 m, 2.25 sec

(d) 1.5 m (e) 4.5 sec (e) 0.4 sec - 4.1 sec

Revision Exercise 8

1. (a) S (0, 0)
(b) Positive quarter turn (90° anticlockwise)
(c) a = 0, b = –1, c = 1, d = 0
2. 1814. 4282 km
3. 12
4. 6
5. < ACB = 90°
6. 36
7. (a) 1.97 (b) 2.95 (c) 1.15
8. (a) 619.8 cm^2 (b) 496.7 cm^3
9. (a) 1 (b) 2
10. (a) $4x^3 - 6x$ (b) $4x^3 + 6x^2 + 2x$ (c) $2x + 7$
(d) $2x$
11. (a) $-x^4 + x^3 + c$ (b) $\frac{1}{3}x^3 + \frac{1}{3x^3} + cx + d$
(c) $\frac{1}{4}x^4 - \frac{1}{3}x^3 + c$ (d) $c - \frac{1}{x} - \frac{1}{2x^2} - \frac{1}{3x^3} - \frac{1}{4x^4}$

12. (a) $\frac{13}{20}$ (b) -1

13. (a) 2.10 m (b) 0.4713 m (c) 12°44′

14. $\mathbf{OX} = \frac{3}{10}\mathbf{a} + \frac{3}{10}\mathbf{c}$

15. (a) $x = -0.6$, $x = -2.6$ (b) $x = 0.8$, $x = -4.8$ (c) $x = 0.8$, $x = 1.28$

(d) $x = -0.3$, $x = 5.8$ (e) $x = 2.6$, $x = 0.4$

Revision Exercise 9

1. 2.35 pm
2. 4 g/cm^3
3. (a)

Class	*Frequency (f)*
1–10	1
11–20	6
21–30	14
31–40	12
41–50	7

(c) Mean = 30

4. (a) $x = 1,\ y = 4$ (b) $x = 6,\ y = 7$

(c) $x = 2,\ y = 0.5$ (d) $x = 9,\ y = -1$

5. (a) Circle, centre O, radius 5 cm

(b) Parallel lines to XY, one on each side, 4 cm apart.

6. A – sh. 600

B – sh. 1 200

C – sh. 400

7. 23.75 units^2

8. (a) $\left(-1, 7\frac{2}{3}\right)$; Maximum; (b) $\left(-2, \frac{-2}{3}\right)$; Inflexion

(b) $\left(1,\ 6\frac{1}{3}\right)$; Minimum

9. (a) $1\frac{1}{3}$ (b) $\frac{-2}{3}$

11. (a) $\frac{5}{2}(x+y)$

(b) (i) $\frac{5}{2}(2x-4)$ or $\frac{5}{2}(2y+4)$ (ii) Perimeter $= 78.4$ cm,

Area = 380. 16 cm^2

12. $<CDE = \frac{1}{2}(7x - y)$

13. 10.4 ± 0.1cm

14. Mean age = 32.79, standard deviation = 21.79

15 (a) On the same axis, draw the graphs of $y = x^3$ and $y = -3x^2 + 27$ point of intersection at $x = 2.4$

(b) On the same axes, draw the graphs of $y = x^3$ and $y = 4x$. Value of x at point of intersection is $x = -2$, 0 and 2.

(c) On the same axes, draw the graphs of $y = x^3$ and $y = 5x + 8$. Value of x at point of intersection is $x = 2.9$.
On the same axes, draw the graphs of $y = x^3$ and $y = 2x^2 - x + 1$. Value of x at point of intersection is $x = 1.6$.

Revision Exercise 10

1. $(x + 3)(2x - 5)$ $x = 2.5$, $x = -3$
2. 10 000
3. 99 : 50
4. (a) 102. 81545 (b) 21.066666 (c) 4897.9592
6. 127
7. $\begin{pmatrix} 3 & 3 & 3 \\ 1 & 12 & -2 \\ 5 & 12 & -1 \end{pmatrix}$ $\begin{pmatrix} 5 & -2 & 3 \\ 2 & 4 & 5 \\ 9 & 3 & 5 \end{pmatrix}$
8. 864
9. $\angle XYZ = 95°$ $\angle YZW = 97.5°$ $\angle ZWX = 70°$ and $\angle WXY = 67.5°$
10. $\mathbf{QM} = \frac{2}{5}(\mathbf{q} - \mathbf{p})$, $\mathbf{QN} = \frac{3}{4}\mathbf{P}$, $\mathbf{NM} = \frac{1}{20}(8\mathbf{q} - 23\mathbf{p})$
11. $E = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ $M = \begin{pmatrix} \frac{-3}{5} & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5} \end{pmatrix}$; $EM = \begin{pmatrix} \frac{9}{5} & \frac{-12}{5} \\ \frac{-12}{5} & \frac{-9}{5} \end{pmatrix}$

$(EM)^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{-4}{15} \\ \frac{-4}{15} & \frac{-1}{5} \end{pmatrix}$

12. (a) -149.4 (b) $-6\frac{2}{3}$

13. 4 trips by 7-ton lorries.
16 trips by 12-ton lorries
14. (a) 16.37 (b) 58.78° (c) 672 cm^3
15. sh. 30893.50 p.m.

Sample Test Paper 1

1. 15   2. 2.412   3. $u = \frac{vf}{f - v}$

4. 1.5   5. $5 - 2\sqrt{6}$ 6. $\frac{-12x}{x^2 - 9}$

7. 9   8. $2y = x - 7$ 9. 2.4

10. Construct a line parallel to AB through C, Q is a point on this line.

11. $14\frac{2}{3}$

12. (i) Equation of tangent is $y = 12x - 14$

(ii) Equation of normal is $y = \frac{-1}{12}x + \frac{61}{6}$

13. $x = -7$

14. (i) $(a - b)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$

(ii) $(0.98)^6 = 0.8858$ (4 s.f.)

15. $\angle ABD = 90°$ ($\angle$ in a semicircle)

$\angle ABD = \angle DBX = 90°$ (angles on a straight line)

$\therefore \angle BDX = 180° - (90° + a°)$ (angle sum of triangle)

$= 90° - a°$

$\angle BDX + \angle BDX = 180°$ (Angles on a straight line)

$\therefore \angle BDC = 180°(90° - a°)$

$= 90° + a°$

16. 30 min

17. (a) Check for correct cumululative frequency table and ogive.

(b) (i) median = 42   (ii) Semi I.R. = 13

(iii) Pass mark = 35.5 %

18. Check for correct table of values for $y = -x^2 + 4x + 1$ and $y = 2x - 3$

Check for correct graph

(a) $x = 0.3$ or 3.7

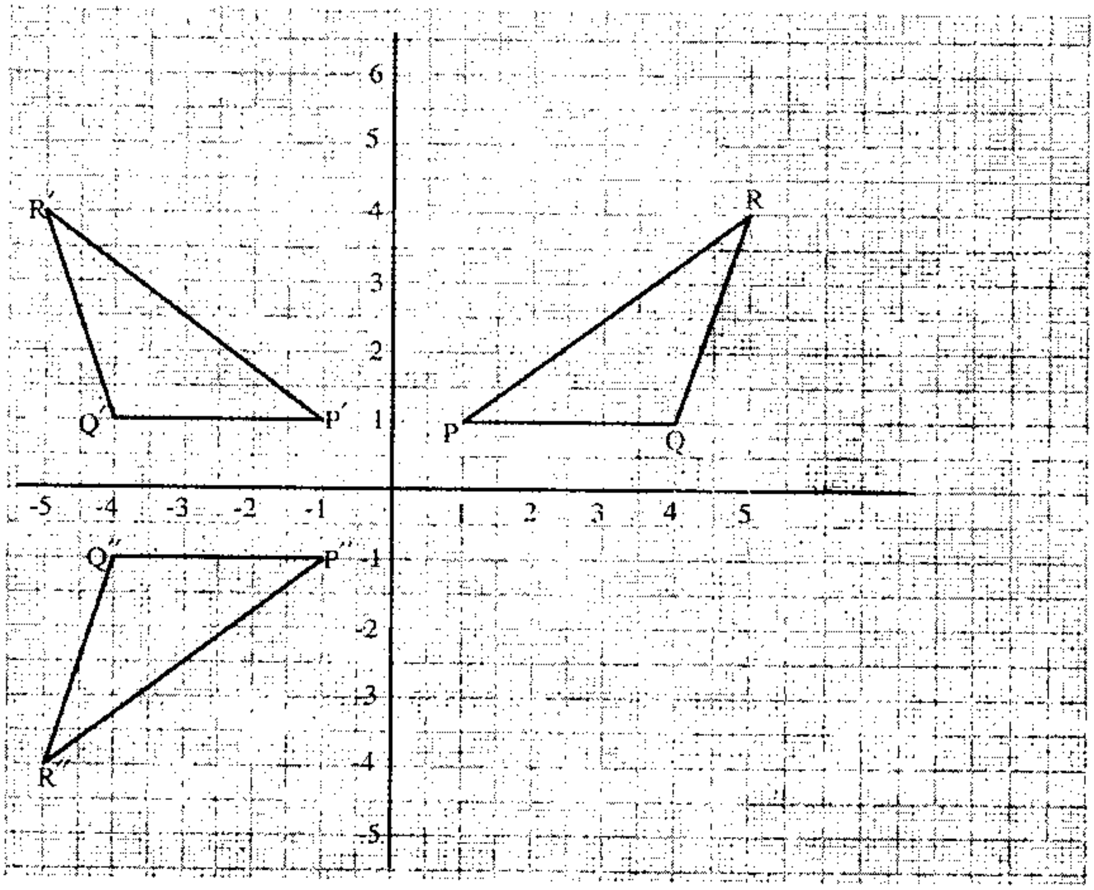
(b) $x = -0.7$ or 2.7

19. (a) 9.125 sq. units (b) 9 sq. units, % error 1.39%.

$$
20.\quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}
\begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 1 & 4 & 5 \\ 1 & 1 & 4 \end{pmatrix} \end{array}
=
\begin{array}{c} \begin{array}{ccc} P' & Q' & R' \end{array} \\ \begin{pmatrix} -1 & -4 & -5 \\ 1 & 1 & 4 \end{pmatrix} \end{array}
$$

$$
\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\begin{array}{c} \begin{array}{ccc} P' & Q' & R' \end{array} \\ \begin{pmatrix} -1 & -4 & -5 \\ 1 & 1 & 4 \end{pmatrix} \end{array}
=
\begin{array}{c} \begin{array}{ccc} P'' & Q'' & R'' \end{array} \\ \begin{pmatrix} -1 & -4 & -5 \\ -1 & -1 & -4 \end{pmatrix} \end{array}
$$

(a)



(b) Rotation of 180° about the origin or enlargement, scale factor −1

21. (a)

x	0	30	60	90	120	150	180	210	240	270	300	330	360
$y = \sin(x + 30°)$	0.05	0.87	1	0.87	0.50	0	−0.50	−0.87	−1.00	−0.87	−0.50	0.00	0.50
$y = 2\cos(x + 30°)$	1.73	1.00	0.00	−1.00	−1.73	−2.00	−1.73	−1.00	0.00	1.00	1.73	2.00	1.73

(b) Check for correct graphs.

(c) $x = 33°$ and $213°$

(d)

Curve	*Amplitude*
$y = \sin(x + 30°)$	1
$y = 2\cos(x + 30°)$	2

22. (a) 90 000 *l* (b) 103.44 m^2 (c) 8.53°
23 (a) Check for correct construction.
(b) Check for correct construction
Radius of the in -circle = 1.5 ± 0.1 cm
(c) A = 14.5 cm^2
24. (a) sh. 4 894 (b) 20 606

Sample Test Paper 2

1. $\frac{6}{5}$
2. $\frac{-4x}{x+5}$
3. $5y - 3x = 18$
4. $\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$
5. 50°
6. 1, 2 and 3
7. 228.6 m
8. 1 700 m*l*
9. 8
10. 600 %
11. $18\frac{3}{4}$ m
12. $186\frac{2}{3}$ cm^2
13. 131.7 km
14. $\frac{25}{56}$
15. 10 km
16. 44°12′′ E
17. (a) (i) 120 000 litres (ii) 22 176 litres (iii) 5 hrs 25 minutes
(b) Ksh. 280 000
18. (a) (i) Ksh.1 600 (ii) 12.23 % (b) 20.36 %
(b) Plan A, by Ksh. 3 600

19. (a) Check for accuracy of graph.
    (b) (i) –4.4 or 0.5
        (ii) $3x^2 + 12x - 7 = 0$
    (c) –4.8 or 0.8.

20. (a) 27.42 cm (b) 77.16 cm (c) 181.8 cm

21. (a)

x	–180	–150	–120	– 90	– 60	– 30	0	30	60	90	120	150	180
2x	–360	–300	–240	–180	–120	– 60	0	60	120	180	240	300	360
3cos x	–3	– 2.6	–1.5	0	1.5	+2.6	3	2.6	1.5	0	–1.5	–2.6	–3
sin 2x	0	0.87	0.87	0	–0.87	–0.87	0	0.87	0.87	0	–0.87	–0.87	0

(b) Check for accuracy of graph.
(c) (i) –90° or 90°
    (ii) 1.1
    (iii) $-60° \le x \le 60°$

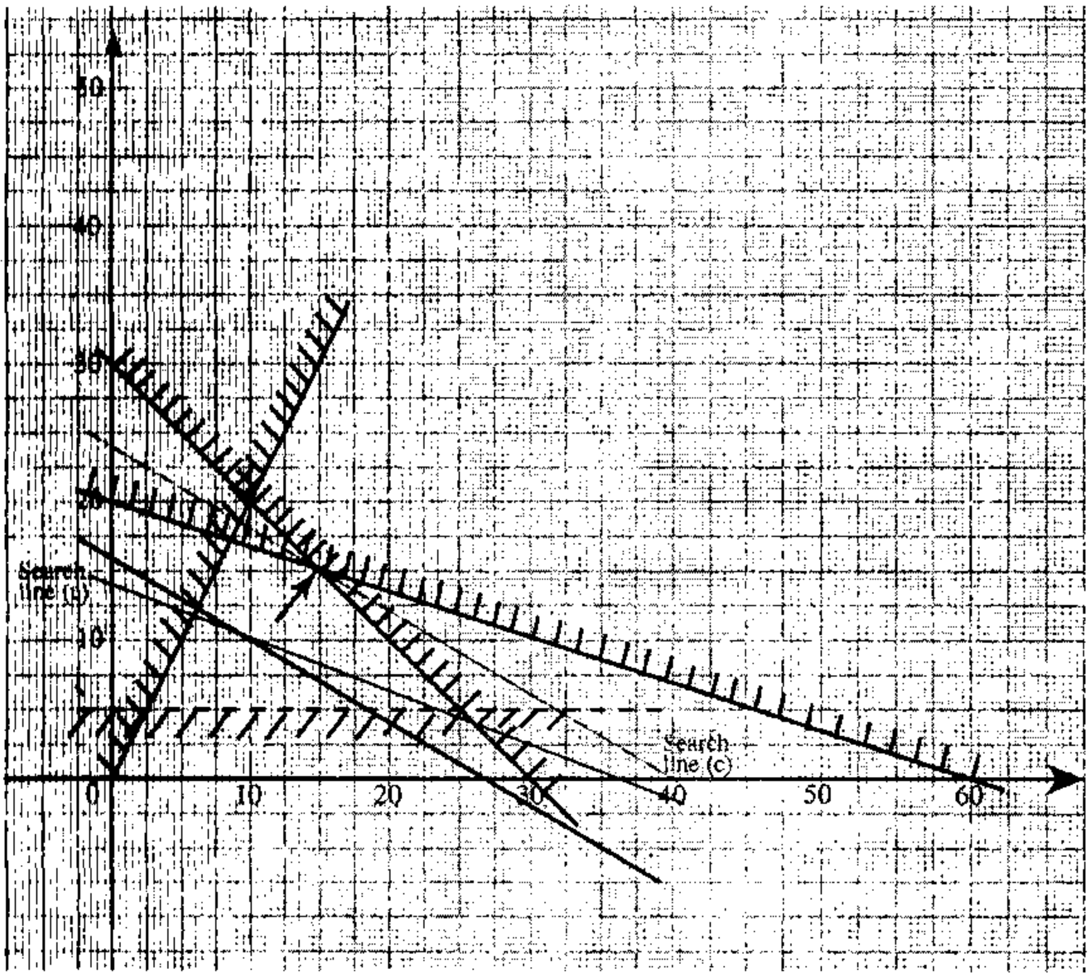
22. (a) 58.57 cm
    (b) (i) 32°46′ (ii) 85°07′ (iii) 52°28′

23. (a)

x	– 4	–3.5	–3	–25	–2	–1.5	–1	–0.5	0	0.5	1
y	24	18.75	14	9.75	6	2.75	0	–2.75	–4	–5.2	6

(b) 38.85 sq. units
(c) $38\frac{5}{6}$ sq.units. 0.05%

24. (a) $x + y \le 30$.............(1)
    $x + 3y \le 60$............(2)
    y + 2x.........................(3)
    y > 5...........................(4)
    600x + 1000y = profit...Objective function



(b) Ksh 24 000

(c) Objective function P = 140x + 350y; Ksh.7 360

Sample Test Paper 3

1. 210.2
2. 5 yrs 8 months
3. $x = \frac{-4}{3}$ or $x = 1$
4. $x = 58°$, $y = 64°$, $z = 58°$
5. 1 403 325 litres
6. $a = +2$ or -2
7. 9 cm
8. $x = \frac{-2(q + 2p)}{Pn - qm}$
9. sh. 3 520
10. (a) $\frac{3}{20}$

    (b) $\frac{19}{40}$
11. L_1; $y + x < 5$, L_2; $2y - x \leq 1$, L_3; $4y + x \geq 8$
12. $-13\frac{1}{4}$
13. $a = 3$, $d = 2$
14. 245.96 cm^2
15. 146 m
16. $x = -1.2$
17. (a) (i) $\frac{2}{5}\mathbf{a} - \mathbf{b}$

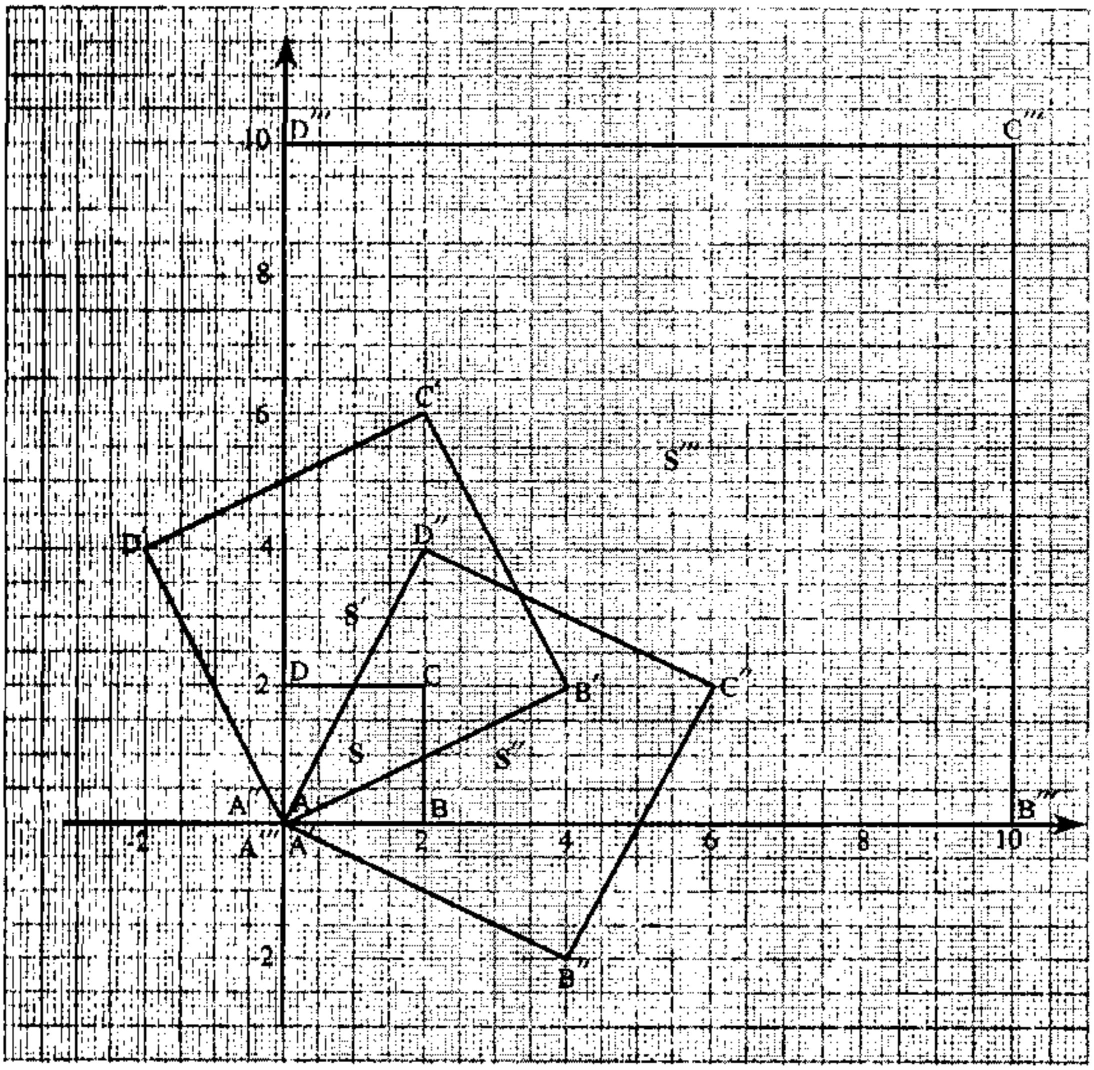
    (ii) $\frac{2}{3}\mathbf{b} - \mathbf{a}$

    (b) $\mathbf{OX} = (1 - t)\mathbf{b} + \frac{2}{5}\mathbf{a}t$

    $\mathbf{OX} = (1 - h)\mathbf{a} + \frac{2}{3}\mathbf{b}h$

    (c) $h = \frac{9}{11}$, $t = \frac{5}{11}$

18. (a)



(b) It is a rotation of –53°.
Centre of rotation is (0, 0)

(c) An enlargement, centre (0, 0), scale factor 5.

19. (a) Check for accuracy of scale drawing.
    (b) (i) N 17° W
        (ii) S 46°W
    (c) (i) 680 km
        (ii) 490 km
    (d) (i) 333° ± 1°
        (ii) 300° ± 1°

20. (a)

x	–6	–5	–4	–3	–2	–1	0	1	2	3	4
y	12	4	–2	–6	–8	–8	–6	–2	4	12	24

    (b) Check for accuracy of drawing.
    (c) (i) x = –4.2 – 1ss and 1.4 – 1ss.
        (ii) x = –3.4 – 1ss and 0.6 – 1ss.

21 (a) 14 cm (b) (i) 6 cm (ii) 10 cm
   (c) 15.17 cm (d) 77.92°

22. (a) Check for accurate construction.
    (b) 3.6 ± 0.1 cm
    (c) 18 cm^2

23. (a) 10.58 cm (b) 34.44 cm^2 (c) 277.18°

24. (a) 29.06 km.
    (b) 10.56 a.m.
    (c) 10.94 km ≃ 10.9 km
    (d) 12.01 p.m.

Sample Test Paper 4

1. 0.0525
2. 16.93 cm
3. (3x –2) (2x –3)
   $x = \frac{2}{3}$ $x = 1\frac{1}{2}$
4. 0. 1539
5. (a) 45°
   (b) Octagon
6. sh. 162857.15
7. Cost of one beaker sh. 120
   Cost of one test tube sh. 50
8. (a) 72°
   (b) 69.3°
9. 33.64 litres
10. a = 9.24 cm
11. $7.5 \le h < 8.5$
12. 51 000
13. $\frac{3}{1\,024}$
14. 5° N 99.21° E

15. $y = 3x^3 + 3$ 16. $P = 110$

17. (a) (i) $y = 2x^2 - 3x - 5$

x	-2	-1	0	1	2	3
y	9	0	-5	-6	-3	4

Check the graph.

(ii) $x = 2.5$

$x = -1$

(b) $y = -2x - 2$

x	0	-1	2
y	-2	0	-6

Check the graph

(c) $x = -1$

$x = 1\frac{1}{2}$

(d) $2x^2 - x - 3 = 0$

18. (a) $\mathbf{OE} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ $\mathbf{AD} = \frac{2}{5}\mathbf{b} - \mathbf{a}$

(b) $s = \frac{4}{7}$, $t = \frac{5}{7}$

(c) $\mathbf{OF} = \frac{4}{7}\mathbf{OE}$ $\mathbf{OF} \uparrow\uparrow \mathbf{OE}$

There is a common point O

Hence O, F and E are collinear.

19. (a) (i) 12868.6 km (ii) 4 800 nm

(b) 88.89 knots

20. (a) (i) 2648.2 m^2 (b) 45.675 m

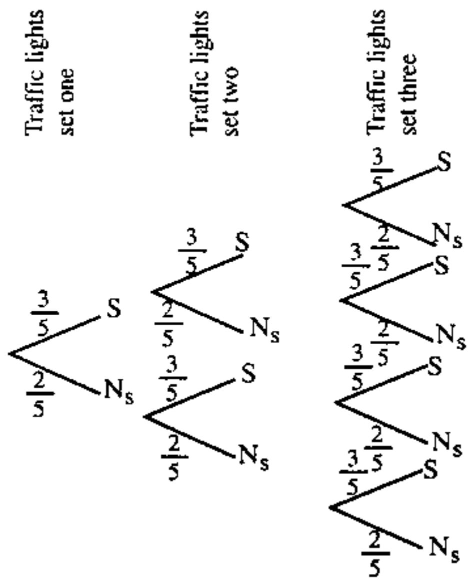
(ii) 66.86°

(iii) 66.2 m

21. (a) (i) 1047.5 (ii) 92.71

(b) 1049.5

22. (a) S represents stopping, Ns represents not stopping.



Possible outcomes

S	S	S
S	S	N
S	N	S
S	N	N
N	S	S
N	S	N
N	N	S
N	N	N

(b) (i) $\frac{27}{125}$ (ii) $\frac{36}{125}$ (iii) $\frac{54}{125}$ (iv) $\frac{8}{125}$

23 (a)

t	2	4	6	8	10	12
v	4	12	28	52	84	124

Total displacement $= 2[4 + 12 + 28 + 52 + 84 + 124]$
$= 2[304]$
$= 608$ m

(b) (i) 610 m (by integration) (ii) 6 ms^{-1}

(c) 0.328 %

24. (a) Check the student's diagram.

(b) (i) Distance SP = 400 km

(ii) Distance SR = 370 km

(iii) Bearing of town 5 from town Q = N43°W or 317°

Sample Test Paper 5

1. -0.4638

2. 1.186

3. 0.009857 mins

4. $x = \frac{-1}{3}$ $\quad x = \frac{2}{3}$

5. (a) $13\frac{1}{3}$ (b) -18.5

6. 1 : 1

7. $\begin{pmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} \end{pmatrix}$ $x = 4, \ y = 3$

8. 6.286

9. $256 - \frac{1024}{x} + \frac{1792}{x^2} - \frac{1792}{x^3} + \frac{1120}{x^4}$

   86.375

10. (3, 5); radius = 2

11. Length = width = 600

12. Increased by 22.5 %

13. 999

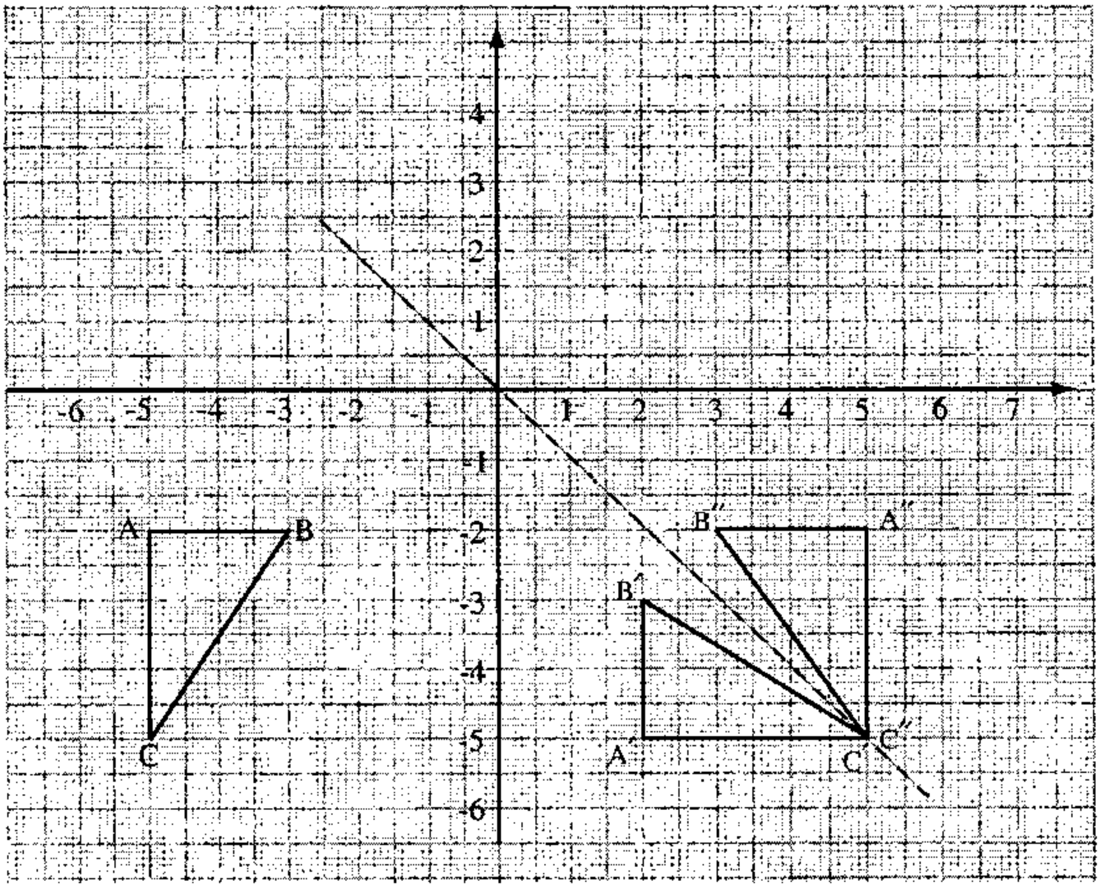
14. 0.004792

15 28

16. $4y \le 20 - 5x$

    $3y \ge 4x - 12, \ y \le 3$

17.



(b) Reflection in the line x = 0.

(c) A´(– 4, –7), B´(– 4,– 5,), C´(– 10, – 10)

18. (a) 473 (b) 718 (c) (i) $2\frac{71}{216}$ (ii) $-3\frac{1}{120}$

19. Check for accuracy of graph.

(i) –3.9, –1.3, 1.3

(ii) –4.7, –0.8, 2.1

(iii) –4, –1.6, 1.8

20.

Class	*x*	*f*	*c.f.*	*fx*	x^2	fx^2
5 – 9	7	4	4	28	49	196
10–19	14.5	12	16	174	210.25	2523
20–39	29.5	8	24	236	870.25	6962
40–49	44.5	8	32	794	1980.25	15842

Mean = 24.8125

s.d. = 27.801

22. $13933\frac{1}{3}$ l; 283106.59cm^2

23. (a) n =1.5; k = 0.158

(b) (i) 189.79

(ii) 185.75

24. (a) $x + 2y \leq 120$

$3x + 2y \leq 180$

$x + y \leq 70$

$x \geq 10$

$y \geq 10$

(c) 40 ha potatoes

30 ha beans

(d) Ksh. 900 000

Sample Test Paper 6

1. -0.4198

2. $\frac{-3}{x+3}$

3. Sally is 18 years old, Tabitha 9 years and Rhoda 12 years old.

4. $2y = 3x + 1$

5. $3\sqrt{5}$

6. $M = \frac{bn}{b+n}$

7. 9182.25

8. 1.5

9. 0.3984 %

10. sh. 10 000

11. 25 %

12. 5 420

13. 45°, 75°

14. 105

15. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

30.4316816

16. $\frac{13}{46}$

17. (b) Half-turn about the origin

18 (a) Median = 45 (b) quartile deviation = 15.75

(c) 40

19. (a)

t	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
v	2	**1.25**	**1**	**1.25**	2	**3.25**	5	**7.25**	**10**	**13.25**	17	**21.25**	**26**	**31.25**	**37**

(b) 78.75

(c) $79\frac{1}{3}$

(d) 1.366 %

20 (a) $\mathbf{AB} = \mathbf{b} - \mathbf{a}$ (ii) $\mathbf{OD} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ (iii) $\mathbf{AE} = \frac{-4}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

(b) 1 : 4

21 (a) $A = 6r + r^2$

(b) 8.1 cm

(c) 6

22 (a) 6 930 l

(b) 15.4 m^2

(c) sh. 3 498

23 (a) 1301.3 km